

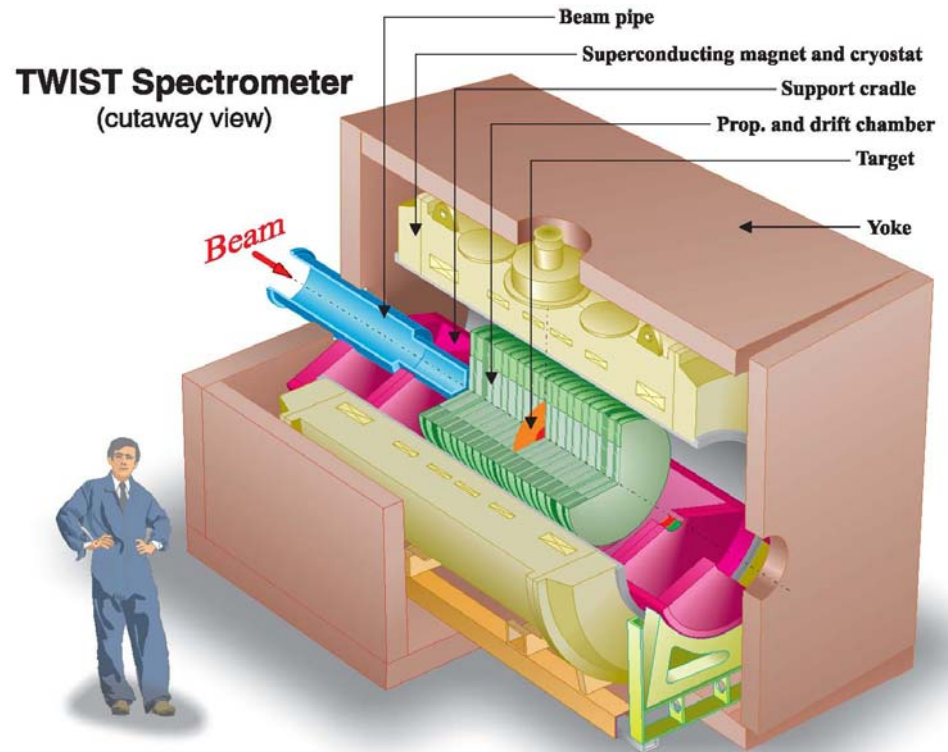
# Measuring muon decay with *TWIST*

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# The TRIUMF Weak Interaction Symmetry Test

- ❑ Tests Standard Model predictions for muon decay.
- ❑ Uses highly polarized  $\mu^+$  beam.
- ❑ Stops  $\mu^+$  in a very symmetric detector.
- ❑ Tracks  $e^+$  through uniform, well-known field.
- ❑ Extracts decay parameters by comparison to detailed and verified simulation.



# Michel parameter description

## □ Muon decay (Michel) parameters $\rho, \eta, \mathcal{P}_\mu, \xi, \delta$

- muon differential decay rate vs. energy and angle:

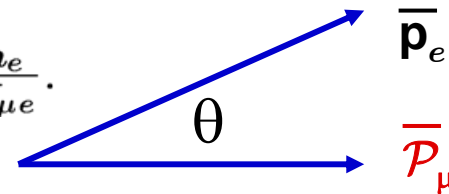
$$\frac{d^2\Gamma}{dx d\cos\theta} = \frac{1}{4}m_\mu W_{\mu e}^4 G_F^2 \sqrt{x^2 - x_0^2} \cdot \{\mathcal{F}_{IS}(x, \rho, \eta) + \mathcal{P}_\mu \cos\theta \cdot \mathcal{F}_{AS}(x, \xi, \delta)\} + R.C.$$

- where

$$\mathcal{F}_{IS}(x, \rho, \eta) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$

$$\mathcal{F}_{AS}(x, \xi, \delta) = \frac{1}{3}\sqrt{x^2 - x_0^2} \left[ \xi \{1-x\} + \frac{2}{3}\xi\delta \left\{4x - 3 + \left(\sqrt{1-x_0^2} - 1\right)\right\} \right]$$

- and  $W_{\mu e} = \frac{m_\mu^2 + m_e^2}{2m_\mu}$ ,  $x = \frac{E_e}{W_{\mu e}}$ ,  $x_0 = \frac{m_e}{W_{\mu e}}$ .



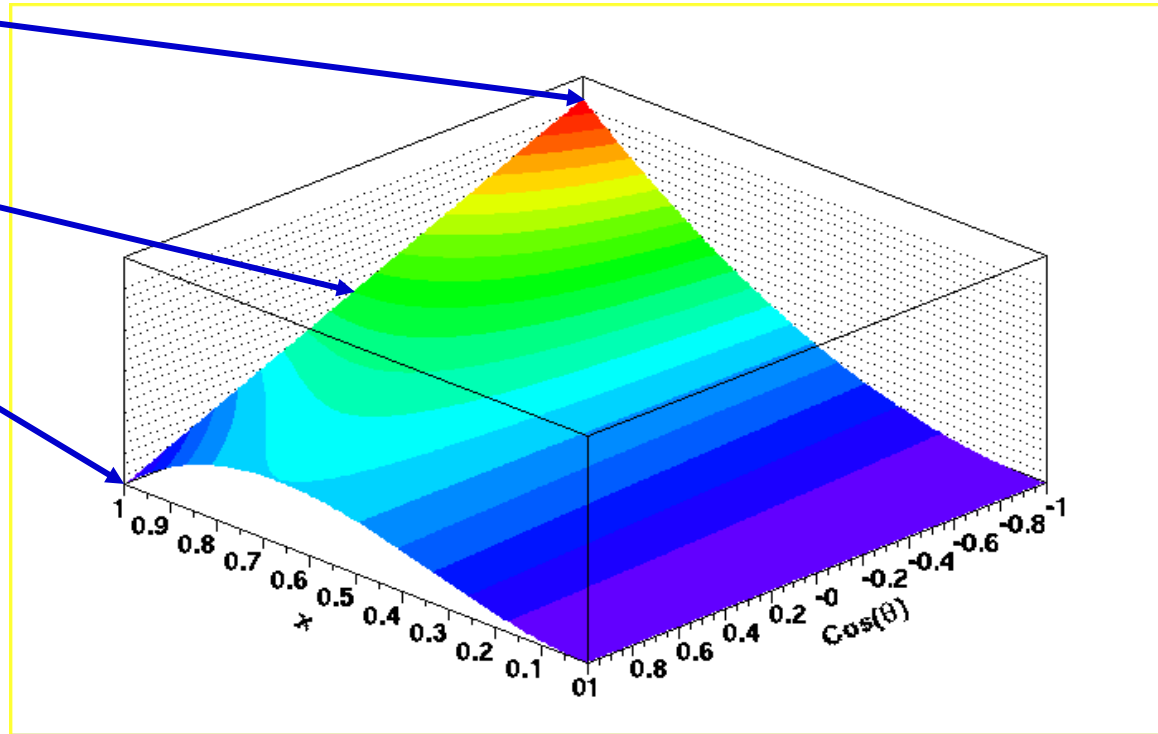
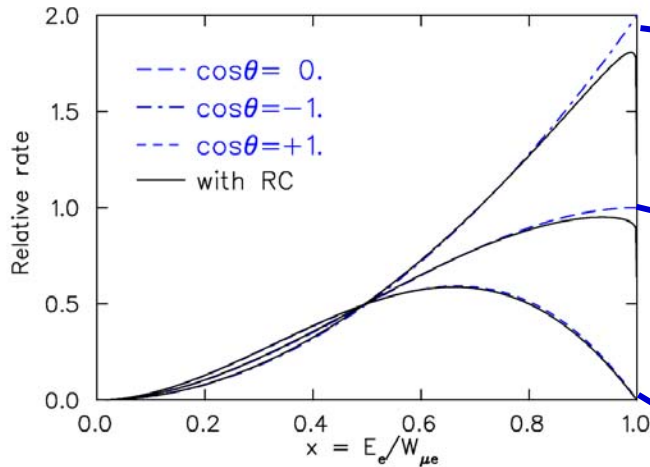
Louis Michel

# Pre-*TWIST* decay parameters

- From the Review of Particle Physics (SM values in parentheses) :
- $\rho = 0.7518 \pm 0.0026$  (Derenzo, 1969) (0.75)
  - $\eta = -0.007 \pm 0.013$  (Burkard *et al.*, 1985) (0.00)
  - $\delta = 0.7486 \pm 0.0026 \pm 0.0028$  (Balke *et al.*, 1988) (0.75)
  - $\mathcal{P}_\mu \xi = 1.0027 \pm 0.0079 \pm 0.0030$  (Beltrami *et al.*, 1987) (1.00)
  - $\mathcal{P}_\mu(\xi\delta/\rho) > 0.99682$  (Jodidio *et al.*, 1986) (1.00)

The goal of *TWIST* is to find any new physics which may become apparent by improving the precision of each of  $\rho$ ,  $\delta$ , and  $\mathcal{P}_\mu \xi$  by at least one order of magnitude compared to prior experimental results.

# Michel shape, graphically



- Full  $O(\alpha)$  radiative corrections with exact electron mass dependence.
- Leading and next-to-leading logarithmic terms of  $O(\alpha^2)$ .
- Leading logarithmic terms of  $O(\alpha^3)$ .
- Corrections for soft pairs, virtual pairs, and an ad-hoc exponentiation.

Arbuzov et al., Phys. Rev. D66 (2002) 93003.  
Arbuzov et al., Phys. Rev. D65 (2002) 113006.

# Michel parameters and coupling constants

□ Fetscher and Gerber coupling constants (see PDG):

$$M = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=R,L}} g_{\varepsilon\mu}^\gamma \langle \bar{e}_\varepsilon | \Gamma^\gamma | (\nu_e)_n \rangle \langle (\bar{\nu}_\mu)_m | \Gamma_\gamma | \mu_\mu \rangle$$

$$\rho = \frac{3}{4} - \frac{3}{4} [ |g_{RL}^V|^2 + |g_{LR}^V|^2 + 2 |g_{RL}^T|^2 + 2 |g_{LR}^T|^2 + \text{Re}(g_{RL}^S g_{RL}^{T*} + g_{LR}^S g_{LR}^{T*}) ]$$

$$\eta = \frac{1}{2} \text{Re}[g_{RR}^V g_{LL}^{S*} + g_{LL}^V g_{RR}^{S*} + g_{RL}^V (g_{LR}^{S*} + 6g_{LR}^{T*}) + g_{LR}^V (g_{RL}^{S*} + 6g_{RL}^{T*})]$$

$$\xi = 1 - \frac{1}{2} |g_{LR}^S|^2 - \frac{1}{2} |g_{RR}^S|^2 - 4 |g_{RL}^V|^2 + 2 |g_{LR}^V|^2 - 2 |g_{RR}^V|^2 + 2 |g_{LR}^T|^2 - 8 |g_{RL}^T|^2 + 4 \text{Re}(g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*})$$

$$\xi\delta = \frac{3}{4} - \frac{3}{8} |g_{RR}^S|^2 - \frac{3}{8} |g_{LR}^S|^2 - \frac{3}{2} |g_{RR}^V|^2 - \frac{3}{4} |g_{RL}^V|^2 - \frac{3}{4} |g_{LR}^V|^2 - \frac{3}{2} |g_{RL}^T|^2 - 3 |g_{LR}^T|^2 + \frac{3}{4} \text{Re}(g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*})$$

# Coupling constants

- Coupling constants  $g_{\varepsilon\mu}^{\gamma}$  can be related to handedness, e.g., total muon right-handed coupling:

$$\begin{aligned}
 Q_R^{\mu} &\equiv Q_{RR} + Q_{LR} \\
 &= \frac{1}{4}|g_{LR}^S|^2 + \frac{1}{4}|g_{RR}^S|^2 + |g_{LR}^V|^2 + |g_{RR}^V|^2 + 3|g_{LR}^T|^2
 \end{aligned}$$

- Global analysis of  $\mu$  decay (Gagliardi *et al.*, PRD **72** 2005)

- no existing similar analysis for other weak decays.

$ g_{RR}^S  < 0.066(0.067)$	$ g_{RR}^V  < 0.033(0.034)$	$ g_{RR}^T  \equiv 0$
$ g_{LR}^S  < 0.125(0.088)$	$ g_{LR}^V  < 0.060(0.036)$	$ g_{LR}^T  < 0.036(0.025)$
$ g_{RL}^S  < 0.424(0.417)$	$ g_{RL}^V  < 0.110(0.104)$	$ g_{RL}^T  < 0.122(0.104)$
$ g_{LL}^S  < 0.550(0.550)$	$ g_{LL}^V  > 0.960(0.960)$	$ g_{LL}^T  \equiv 0$

- Neutrino mass implications at  $10^{-7}$ - $10^{-4}$  for LR/RL:

- J. Kile (CIPANP06), Erwin *et al.* (hep-ph/0602240)

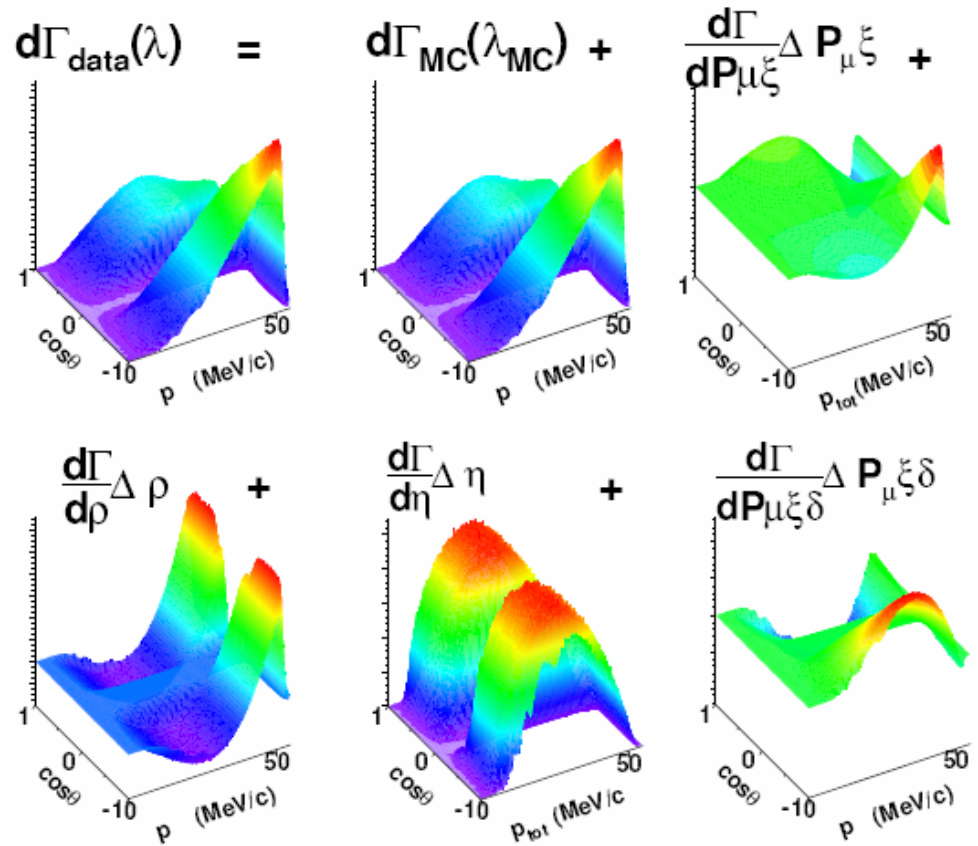
# Fitting the data distributions

□ Michel distribution is linear in  $\rho$ ,  $\eta$ ,  $\mathcal{P}_{\mu\xi}$ , and  $\mathcal{P}_{\mu\xi\delta}$ , so a fit to first order expansion is exact.

□ Fit data to simulated (MC) base distribution with *hidden assumed parameters*,

$$\lambda_{\text{MC}} = (\rho, \eta, \mathcal{P}_{\mu\xi} | \mathcal{P}_{\mu\xi\delta}, \mathcal{P}_{\mu\xi\delta})$$

plus MC-generated distributions from analytic derivatives, times fitting parameters ( $\Delta\lambda$ ) representing deviations from base MC.



(graphic thanks to Blair Jamieson)



# Evaluation of Systematic Uncertainties

- *TWIST* relies on a fit to simulation:
  - Simulation must be verified.
  - Reconstruction systematics eliminated if simulation is perfect.

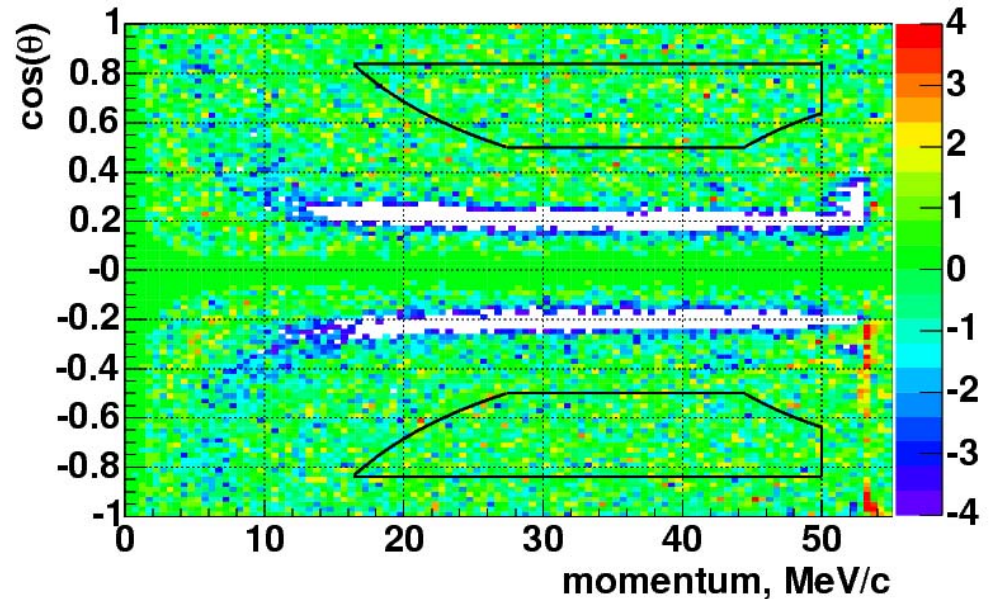
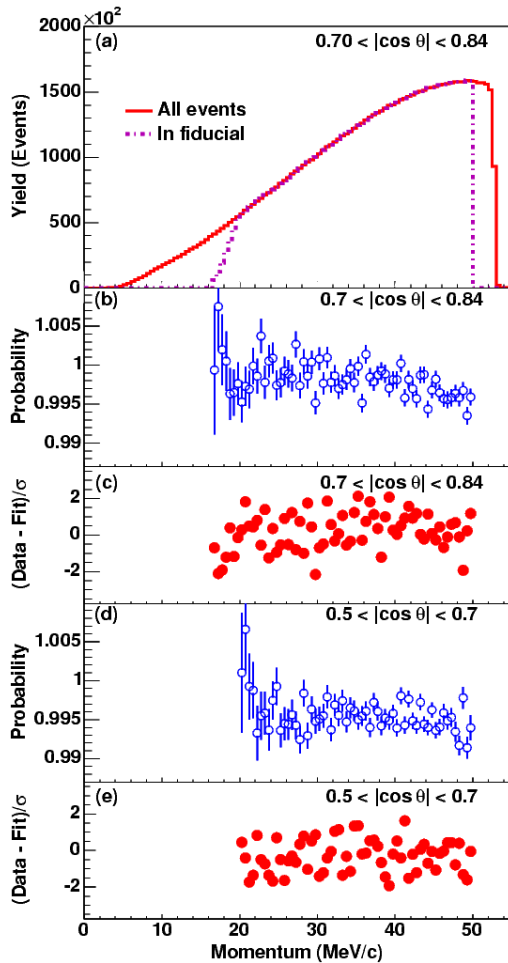
- General method:
  - exaggerate a condition (in data or MC) which may cause error.
  - measure effect by fitting, using correlated sets where practical.
  - scale results according to variance in a data set.
  - Linearity? Double counting?

- Positron interactions:
  - Energy smearing
  - Multiple scattering
  - Hard interactions
  - Material in detector
  - Material outside

- Chamber response:
  - DC and PC efficiencies
  - Dead zone
  - Long drift times
  - HV variations
  - Temperature, pressure
  - Chamber foil bulges
  - Crosstalk
  - Variation of  $t_0$

- Momentum calibration:
  - End point fits
  - Field reproduction
- Muon beam stability:
  - Stopping location
  - Beam intensity
  - Magnet stability
- Spectrometer alignment:
  - Translations
  - Rotations
  - Longitudinal
  - Field to detector axis

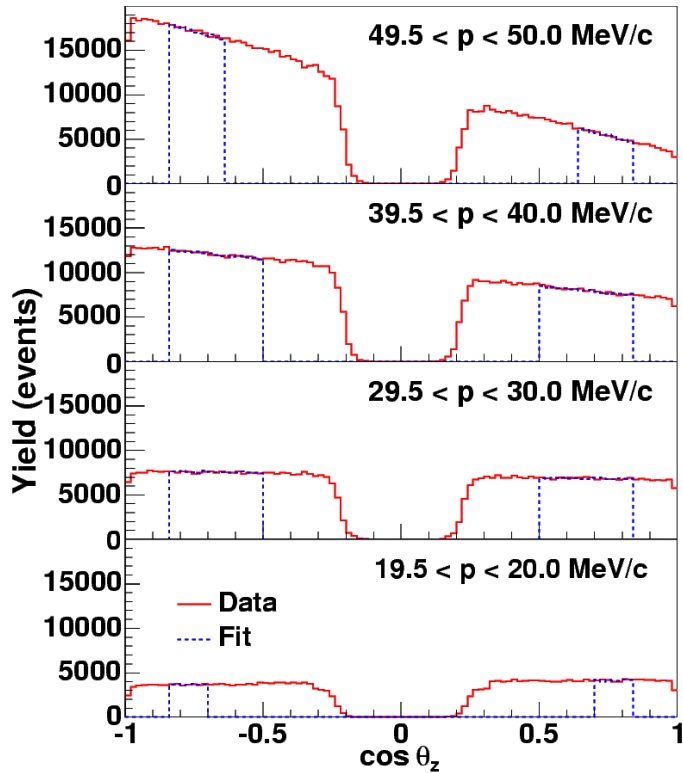
# Fits to data distributions



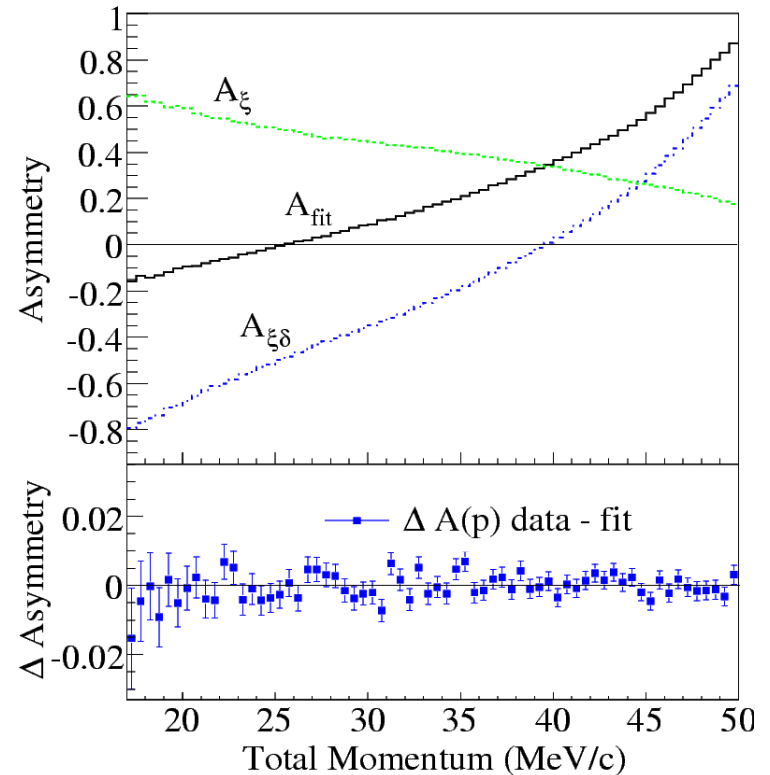
Above: normalized residuals of fit, and fiducial region used for fit:  $p < 50$  MeV/c,  $0.50 < |\cos \theta| < 0.84$ ,  $|p_z| > 13.7$  MeV/c,  $p_T < 38.5$  MeV/c.

Left: comparison of data to fit (MC) vs. momentum, also showing (MC reconstructed)/(MC thrown) comparisons and normalized residuals.

# Fits to data distributions (cont.)



Angular distributions for restricted momentum ranges. Dashed lines show fiducial region of two-dimensional fit.



Dependence of asymmetry on momentum, its two contributions, and comparison of data and fit (MC) distributions.

# Summary of results: $\rho$ and $\delta$

- $\rho = 0.75080 \pm 0.00044(\text{stat}) \pm 0.00093(\text{syst}) \pm 0.00023(\eta)$ 
  - 2.5 times better precision than PDG value.
  - Uncertainty scaled for  $\chi^2/\text{dof} = 7.5/4$  (CL=0.11) for different data sets.
  - J.R. Musser *et al.*, PRL **94**, 101805 (2005), hep-ex/0409063.
  
- $\delta = 0.74964 \pm 0.00066(\text{stat}) \pm 0.00112(\text{syst})$ 
  - 2.9 times better precision than PDG value.
  - A. Gaponeko *et al.*, PRD **71**, 071101(R) (2005), hep-ex/0410045.
  
- Using the above values of  $\rho$  and  $\delta$ , with  $\mathcal{P}_\mu(\xi\delta/\rho) > 0.99682$  (PDG) and  $Q_R^\mu \geq 0$ , we get
$$0.9960 < \mathcal{P}_\mu \xi \leq \xi < 1.0040 \text{ (90\% c.l.)}$$
  - improves upon  $\mathcal{P}_\mu \xi = 1.0027 \pm 0.0079 \pm 0.0030$ .

# Systematic uncertainties: $\rho$ and $\delta$

Systematic uncertainties	$\rho$ ( $\times 10^4$ )		$\delta$ ( $\times 10^4$ )	
	published	current	published	current
Chamber response (ave)	5.1	0.1	5.6	1.5
Stopping target thickness	4.9	-	3.7	-
Positron interactions	4.6	3.6	5.5	6.5
Spectrometer alignment	2.2	0.4	6.1	-
Momentum calibration (ave)	2.0	1.1	2.9	2.2
Theoretical radiative correction	2.0	2.0	1.0	1.0
Muon beam stability (ave)	0.4	0.5	1.0	0.9
Track selection algorithm	1.1	-		
Asymmetric efficiencies			0.4	0.1
<b>Total in quadrature</b>	<b>9.3</b>	<b>4.3</b>	<b>11.2</b>	<b>7.2</b>

New data and analysis: thesis of R.P. MacDonald, in preparation.

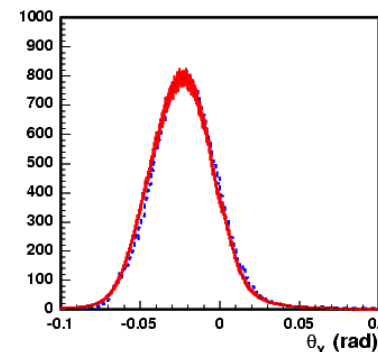
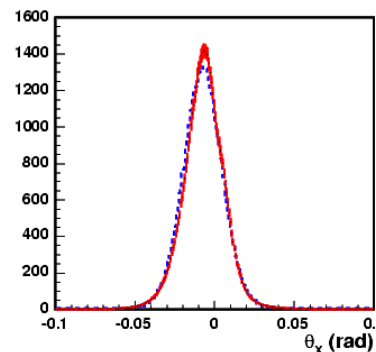
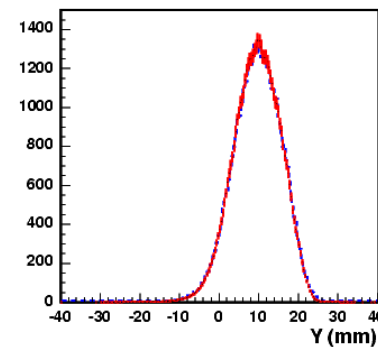
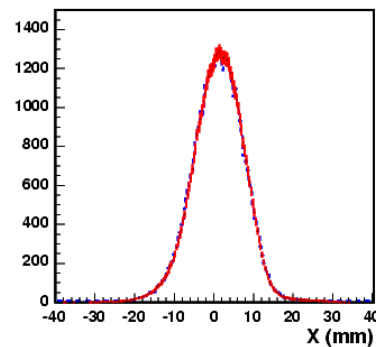
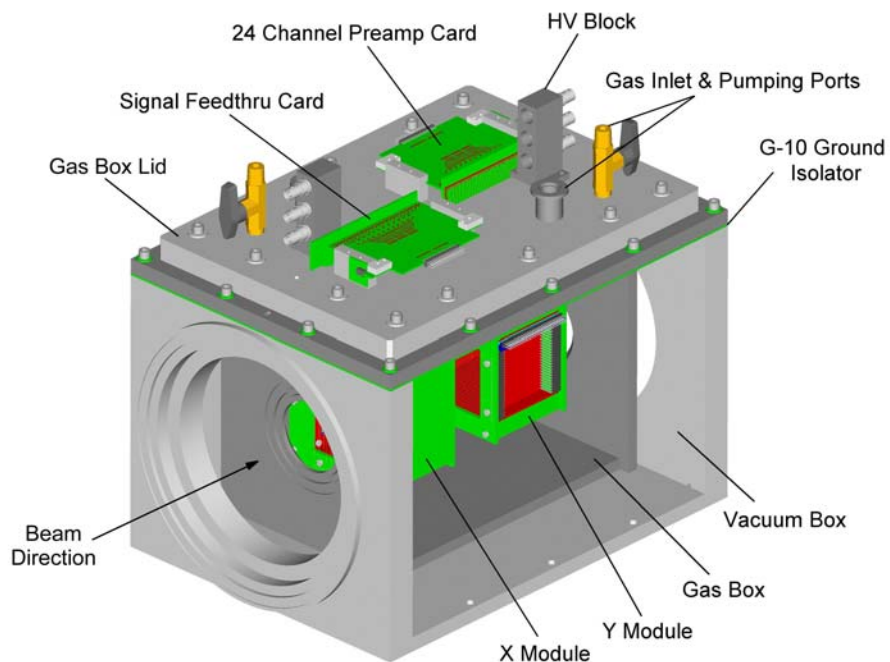
# Summary of results: $\mathcal{P}_\mu^\xi$

- $\mathcal{P}_\mu^\xi = 1.0003 \pm 0.0006(\text{stat}) \pm 0.0038(\text{syst})$ 
  - 2.2 times better precision than PDG value (Beltrami *et al.*).
  - still not as precise as TWIST indirect result from  $\rho$  and  $\delta$ .
  - B. Jamieson *et al.*, recently submitted to PRD, hep-ex/0605100.
  
- Dominated by systematic uncertainty from spectrometer fringe field depolarization:
  - prospects for improvement are excellent.
  - data was taken in 2004; new data with improved muon beam will be taken in 2006-07.

# Systematic uncertainties: $\mathcal{P}_\mu \xi$

Systematic uncertainties	$\mathcal{P}_\mu \xi (\times 10^3)$
Depolarization in fringe field (ave)	3.4
Depolarization in muon stopping material (ave)	1.2
Chamber response (ave)	1.0
Spectrometer alignment	0.3
Positron interactions (ave)	0.3
Depolarization in muon production target	0.2
Momentum calibration	0.2
Upstream-downstream efficiency	0.2
Background muon contamination (ave)	0.2
Beam intensity (ave)	0.2
Michel $\eta$ parameter	0.1
Theoretical radiative correction	0.1
<b>Total in quadrature</b>	<b>3.8</b>

# Fringe field systematic improvement



The TECs (time expansion chambers) are transverse drift chambers operating at 0.08 bar, separated from beam vacuum by 6  $\mu\text{m}$  Mylar windows. Two modules measure x and y.



# Left-right symmetric models

- Weak eigenstates in terms of mass eigenstates and mixing angle:

$$W_L = W_1 \cos \zeta + W_2 \sin \zeta, \quad W_R = e^{i\omega} (-W_1 \sin \zeta + W_2 \cos \zeta)$$

- Assume possible differences in left and right couplings and CKM character.

Use notation: 
$$t = \frac{g_R^2 m_1^2}{g_L^2 m_2^2}, \quad t_\theta = t \frac{|V_{ud}^R|}{|V_{ud}^L|}, \quad \zeta_g = \frac{g_R^2}{g_L^2}$$

- Then, for muon decay, the Michel parameters are modified:

$$\rho = \frac{3}{4}(1 - 2\zeta_g^2), \quad \xi = 1 - 2(t^2 + \zeta_g^2),$$

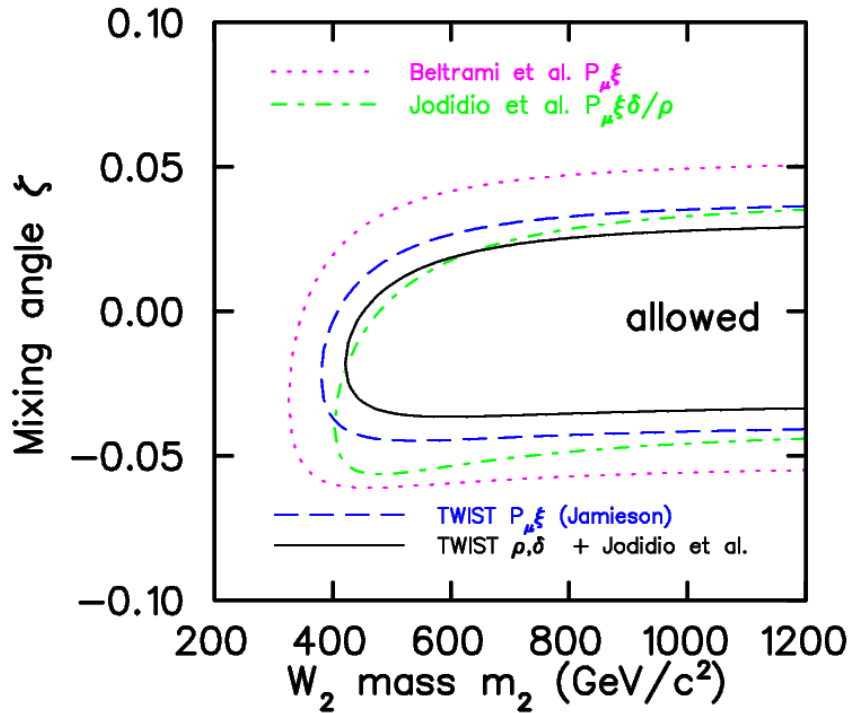
$$\mathcal{P}_\mu = 1 - 2t_\theta^2 - 2\zeta_g^2 - 4t_\theta\zeta_g^2 \cos(\alpha + \omega)$$

- “manifest” LRS assumes  $g_R = g_L$ ,  $V^R = V^L$ ,  $\omega = 0$  (no CP violation).
  - “pseudo-manifest” LRS allows CP violation, but  $V^R = (V^L)^*$  and  $g_R = g_L$ .
  - RS “non-manifest” or generalized LRS makes no such assumptions.
- Most experiments must make assumptions about LRS models!**

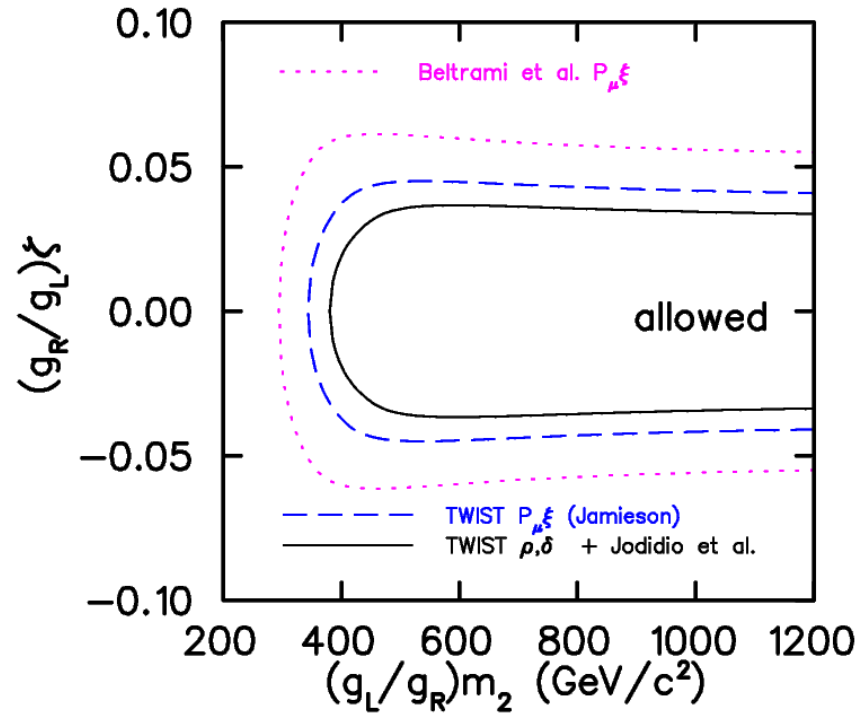
# Limits on LRS parameters: PDG04

Observable	$m_2$ (GeV/c <sub>2</sub> )	$ \zeta $	+	-
$m(K_L - K_S)$	>1600		reach	(P)MLRS
Direct $W_L$ searches	>720-650 (D0) >652 (CDF)		clear signal	(P)MLRS decay model
CKM unitarity		$<10^{-3}$	sensitivity	(P)MLRS heavy $\nu_R$
$\beta$ decay	>310	$<0.040$	both parameters	(P)MLRS light $\nu_R$
$\mu$ decay ( <i>TWIST</i> )	>406 (>420)	$<0.033$ ( $<0.030$ )	model independence	light $\nu_R$

# Muon decay LRS limits



Restricted ("manifest") LRS model



General LRS model

**Exclusion (90% cl) plots for left-right symmetric model mixing angle and right partner boson  $W_2$  mass  $m_2$**

# Summary

- *TWIST* has produced its first direct measurement of  $\mathcal{P}_\mu\xi$ , to add to previous results for  $\rho$  and  $\delta$ .
- Analysis underway for second measurements for  $\rho$  and  $\delta$ , representing further improvements by  $\sim 2$ .
- Reduction of depolarization systematics for  $\mathcal{P}_\mu\xi$  seems achievable, but it is not yet known by how much.
- In 2006-2008, *TWIST* will produce its final results:
  - goal remains the reduction of uncertainty by an order of magnitude compared to previous muon decay parameter experiments.

# *TWIST* Participants

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