Latest Results on ρ and δ from Muon Decay

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for the ***TWIST* Collaboration**

Muon decay spectrum

The energy and angle distributions of positrons following polarized muon decay obey the spectrum:

$$\frac{d^2\Gamma}{x^2 dx d(\cos\theta)} \propto (3-3x) + \frac{2}{3}\rho(4x-3) + 3\eta\frac{x_0}{x}(1-x)$$

$$+ P_\mu \xi \cos\theta\left[(1-x) + \frac{2}{3}\delta(4x-3)\right]$$

where $x = \frac{E_e}{E_{e,\max}}$

[Radiative corrections not included]

Muon decay matrix element

- Most general local, derivative-free, lepton-number conserving muon decay matrix element:

$$M = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=R,L}} g^{\gamma}_{\varepsilon\mu} \left\langle \bar{e}_{\varepsilon} \,|\, \Gamma^{\gamma} \,|\, (\nu_e)_n \right\rangle \left\langle (\bar{\nu}_{\mu})_m \,|\, \Gamma_{\gamma} \,|\, \mu_{\mu} \right\rangle$$

- In the Standard Model, g^{V}_{LL} = 1, all others are zero
- Pre-*TWIST* global fit results (all 90% c.l.):

$\lvert g^S_{RR} \rvert < 0.066$	$\lvert g^V_{RR} \rvert < 0.033$	$\lvert g^T_{RR} \rvert \equiv 0$
$\lvert g^S_{LR} \rvert < 0.125$	$\lvert g^V_{LR} \rvert < 0.060$	$\lvert g^T_{LR} \rvert < 0.036$
$\lvert g^S_{RL} \rvert < 0.424$	$\lvert g^V_{RL} \rvert < 0.110$	$\lvert g^T_{RL} \rvert < 0.122$
$\lvert g^S_{LL} \rvert < 0.550$	$\lvert g^V_{LL} \rvert > 0.960$	$\lvert g^T_{LL} \rvert \equiv 0$

Muon decay parameters and coupling constants

$$\rho = \frac{3}{4} - \frac{3}{4}[\left|g^V_{RL}\right|^2 + \left|g^V_{LR}\right|^2 + 2\left|g^T_{RL}\right|^2 + 2\left|g^T_{LR}\right|^2 + \mathbb{R}e\,(g^S_{RL}g^{T*}_{RL} + g^S_{LR}g^{T*}_{LR})]$$

$$\eta = \frac{1}{2}\mathbb{R}e[g^V_{RR}g^{S*}_{LL} + g^V_{LL}g^{S*}_{RR} + g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR}) + g^V_{LR}(g^{S*}_{RL} + 6g^{T*}_{RL})]$$

$$\xi = 1 - \frac{1}{2}\left|g^S_{LR}\right|^2 - \frac{1}{2}\left|g^S_{RR}\right|^2 - 4\left|g^V_{RL}\right|^2 + 2\left|g^V_{LR}\right|^2 - 2\left|g^V_{RR}\right|^2 + 2\left|g^T_{LR}\right|^2 - 8\left|g^T_{RL}\right|^2 + 4\mathbb{R}e(g^S_{LR}g^{T*}_{LR} - g^S_{RL}g^{T*}_{RL})$$

$$\xi\delta = \frac{3}{4} - \frac{3}{8}\left|g^S_{RR}\right|^2 - \frac{3}{8}\left|g^S_{LR}\right|^2 - \frac{3}{2}\left|g^V_{RR}\right|^2 - \frac{3}{4}\left|g^V_{RL}\right|^2 - \frac{3}{4}\left|g^V_{LR}\right|^2 - \frac{3}{2}\left|g^T_{RL}\right|^2 - 3\left|g^T_{LR}\right|^2 + \frac{3}{4}\mathbb{R}e(g^S_{LR}g^{T*}_{LR} - g^S_{RL}g^{T*}_{RL})$$

Prior to ***TWIST***

	SM
ρ = 0.7518 ± 0.0026	3/4
η = -0.007 ± 0.013	0
$P_\mu\xi$ = 1.0027 ± 0.0079 ± 0.0030	1
δ = 0.7486 ± 0.0026 ± 0.0028	3/4
$P_\mu(\xi\delta/\rho)$ > 0.99682 (90% c.l.)	1

Goal of *TWIST*

- Search for new physics that can be revealed by **order-of-magnitude improvements** in our knowledge of **ρ**, **δ**, and ***P*$_\mu$ξ**

Two examples

- Model-independent limit on muon handedness

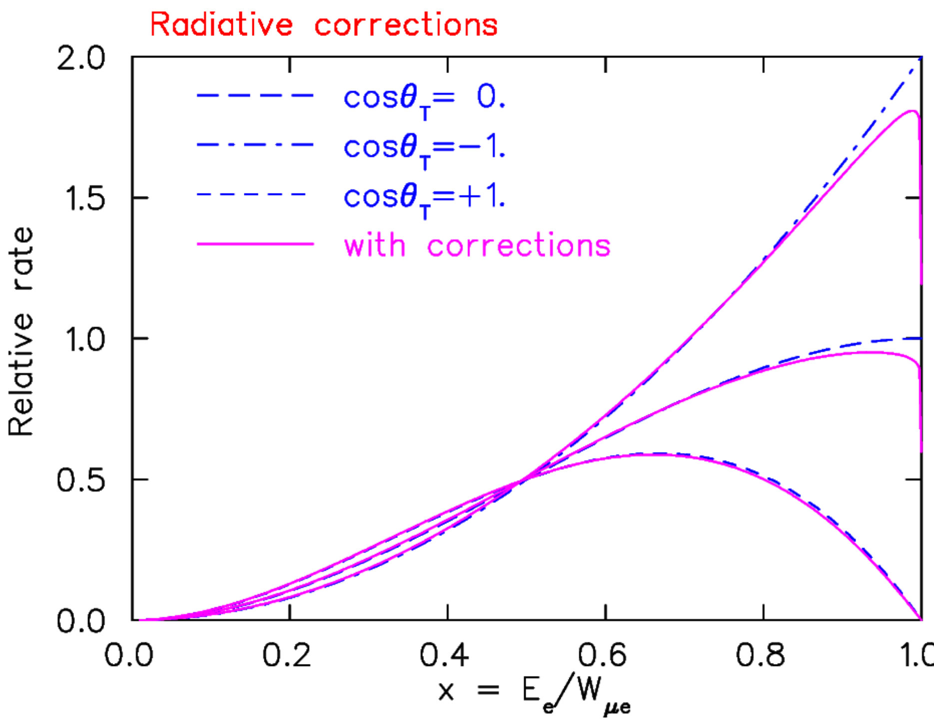
$$Q_R^\mu = \frac{1}{2}\left[1 + \frac{1}{3}\xi - \frac{16}{9}\xi\delta\right]$$

- Left-right symmetric models

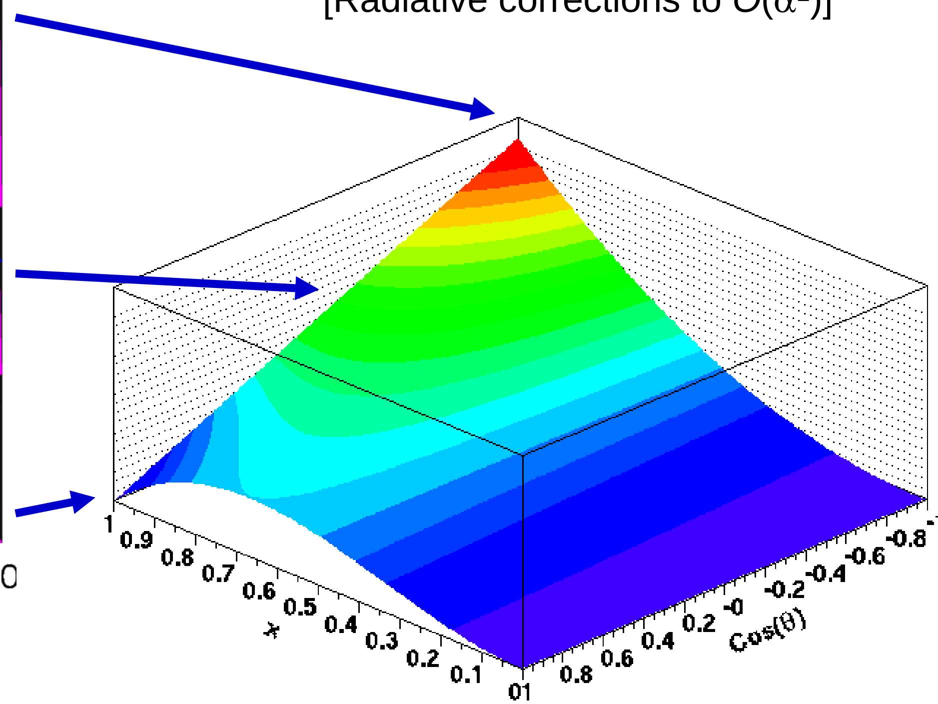
$$\frac{3}{4} - \rho = \frac{3}{2}\zeta^2 \qquad 1 - P_\mu\xi = 4\left(\zeta^2 + \zeta\left(\frac{M_L}{M_R}\right)^2 + \left(\frac{M_L}{M_R}\right)^4\right)$$

- …..

What is required?



[Radiative corrections to $O(\alpha^2)$]

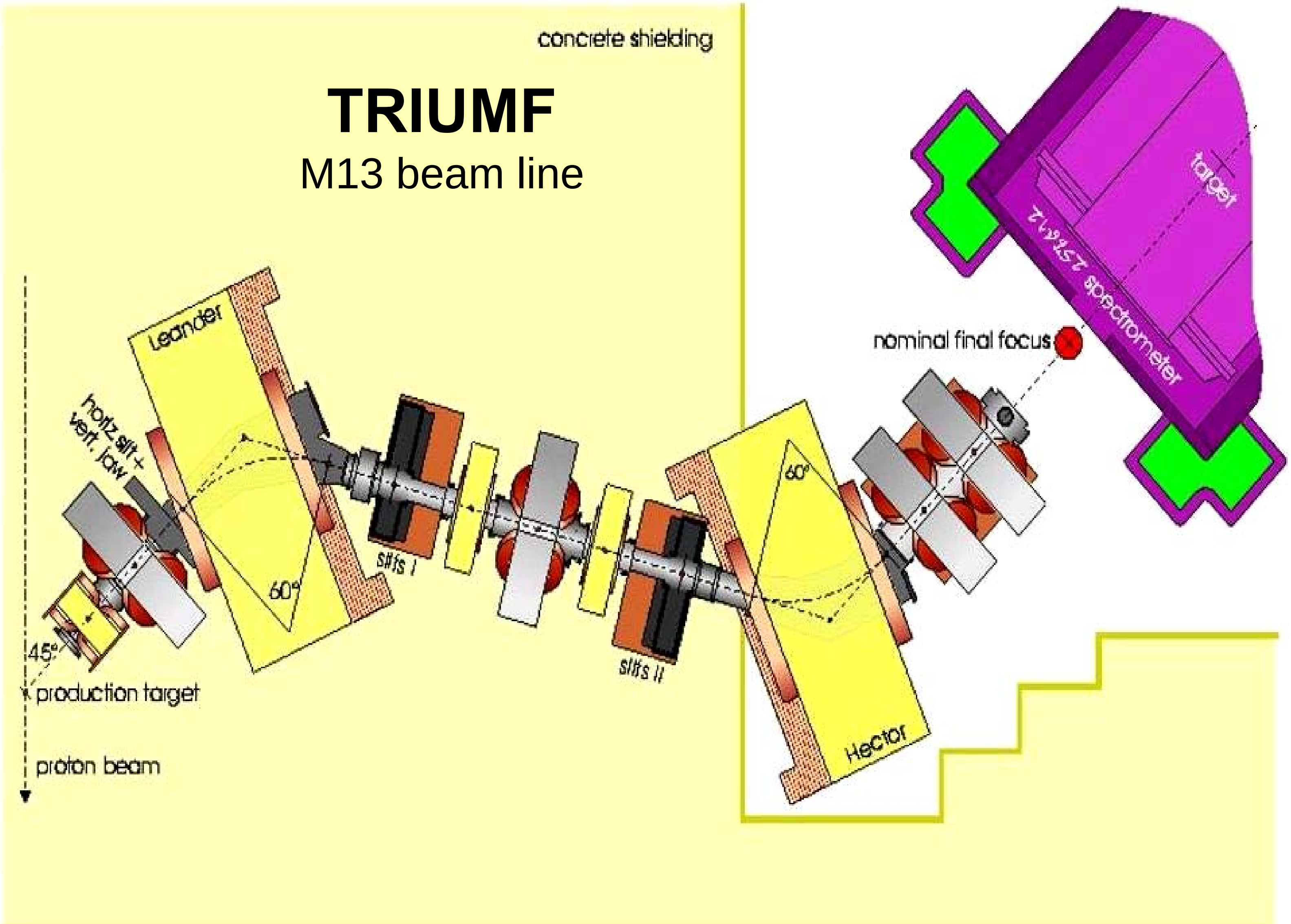


Must:

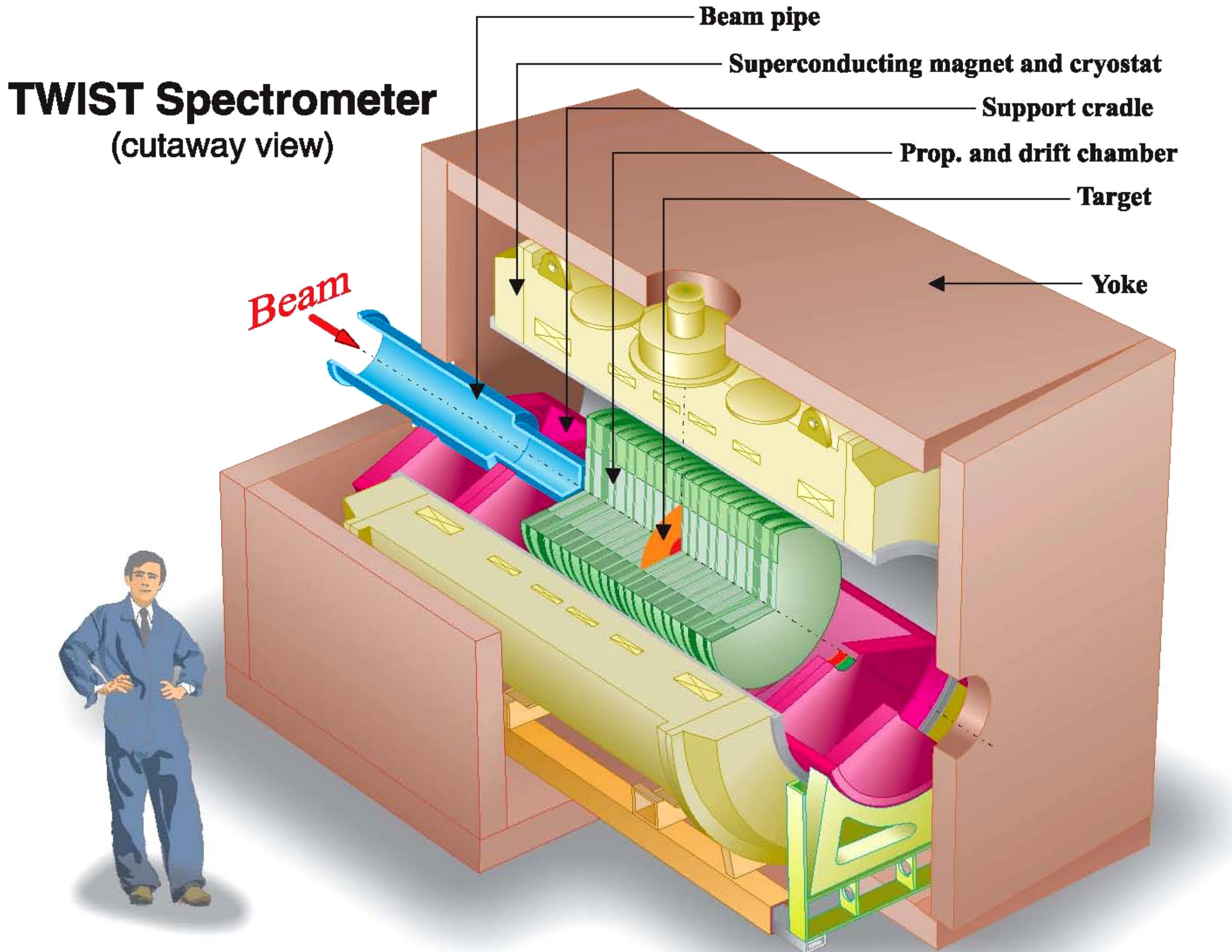
- Understand sources of muon depolarization
  - -- P_μ and **ξ** come as a product
- Determine spectrum shape
  - -- All three parameters
- Measure forward-backward asymmetry
  - -- For $P_\mu \xi$ and **δ**

to within a **few parts in 10^4**

Surface muon beam

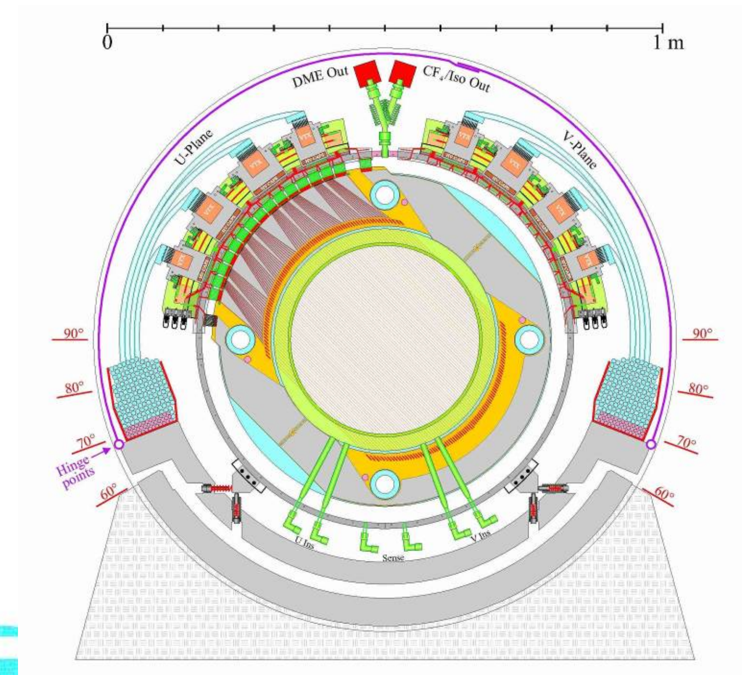


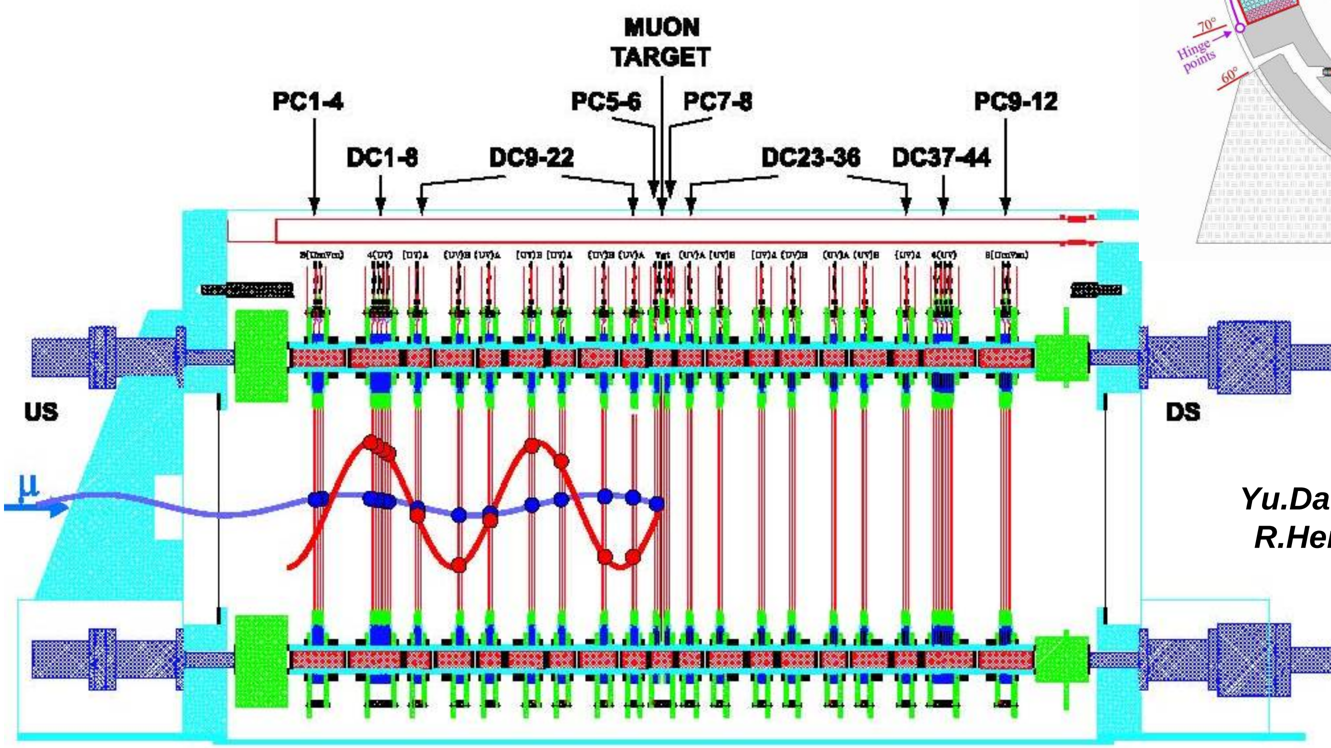
TWIST spectrometer



Detector array

- 56 low-mass high-precision planar chambers symmetrically placed around thin target foil (DME,CF_4/Isobutane)
- Measurement initiated by single thin scintillation counter at entrance to detector
- Beam stop position controlled by variable He/CO_2 gas degrader

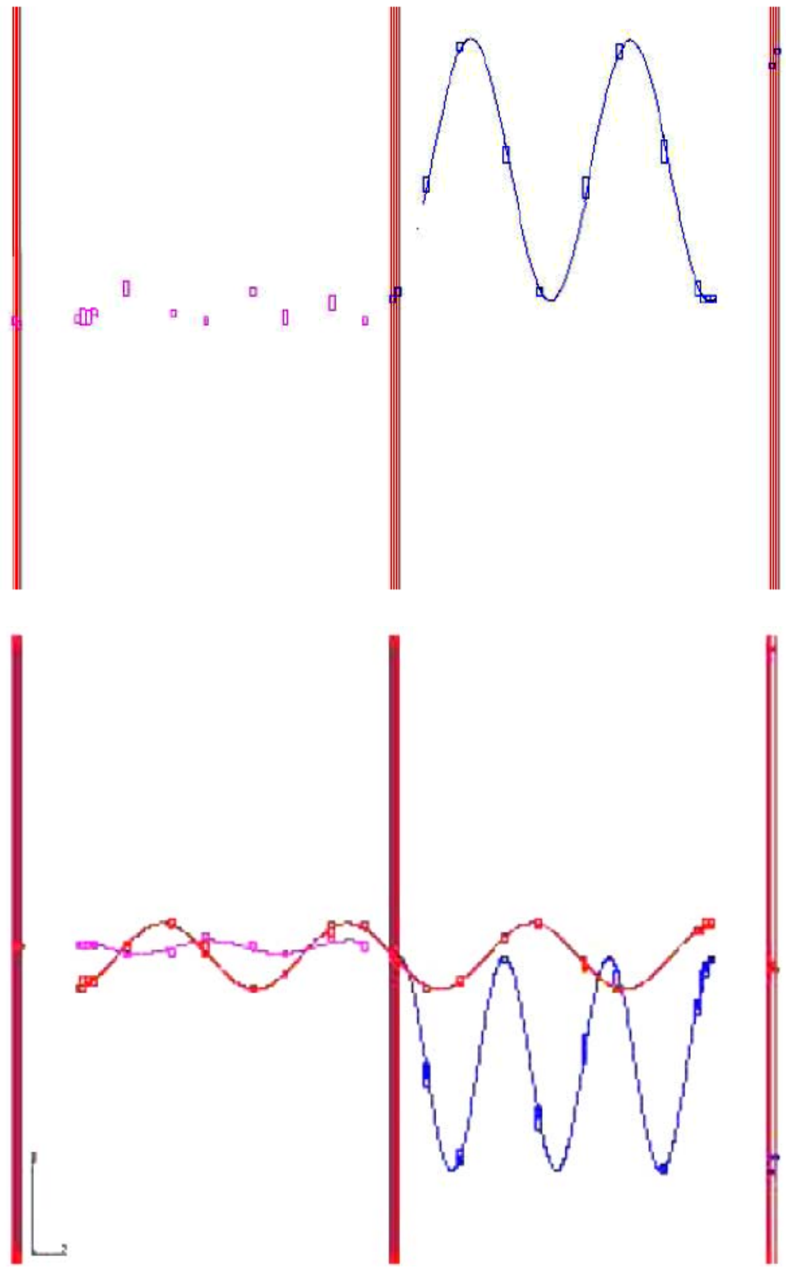




Yu.Davydov et al. NIM A461(2001)68
R.Henderson et al. NIM A548(2005)306

Typical events

- Use pattern recognition (in position and time) to sort hits into tracks, then fit to helix



- Must also recognize beam positrons, delta tracks, backscattering tracks

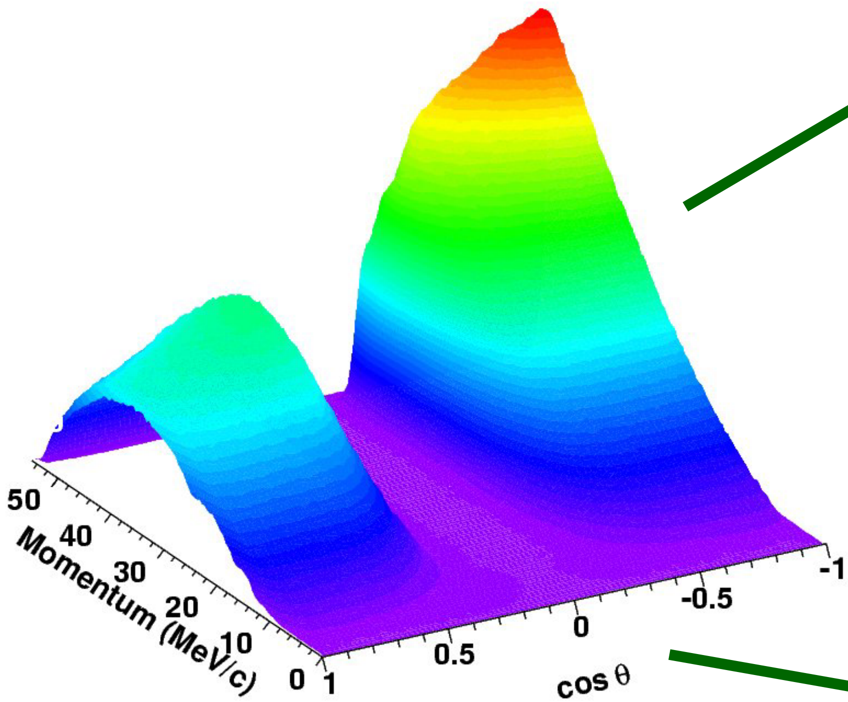
Physics data sets

- Fall 2002
  - Test data-taking procedures and develop analysis techniques
  - First physics results – ρ and δ
  - Graphite-coated Mylar target not suitable for $P_\mu\xi$
- Fall 2004
  - Al target (70 μm) and Time Expansion Chamber enabled first $P_\mu\xi$ measurement
  - Improved determinations of ρ and δ recently published
- 2006-07
  - Ag and Al target data
  - Larger data sets and better beam characterization
  - Achieve ultimate ***TWIST*** precision for ρ, δ, and $P_\mu\xi$

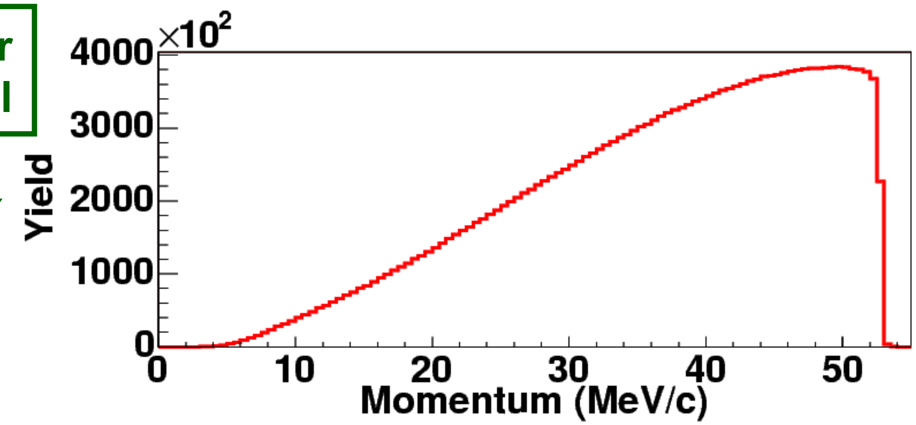
Analysis method

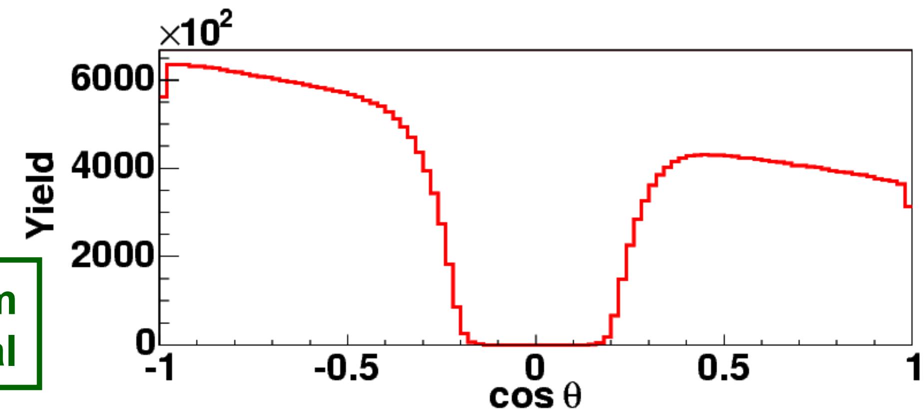
- Extract energy and angle distributions for data:
  - Apply (unbiased) cuts on muon variables.
  - Reject fast decays and backgrounds.
  - Calibrate e^+ energy to kinematic end point at 52.83 MeV.
- Fit to identically derived distributions from simulation:
  - GEANT3 geometry contains virtually all detector components.
  - Simulate chamber response in detail.
  - Realistic, measured beam profile and divergence.
  - Extra muon and beam positron contamination included.
  - Output in digitized format, identical to real data.

2-d momentum-angle spectrum



In angular fiducial





In momentum fiducial

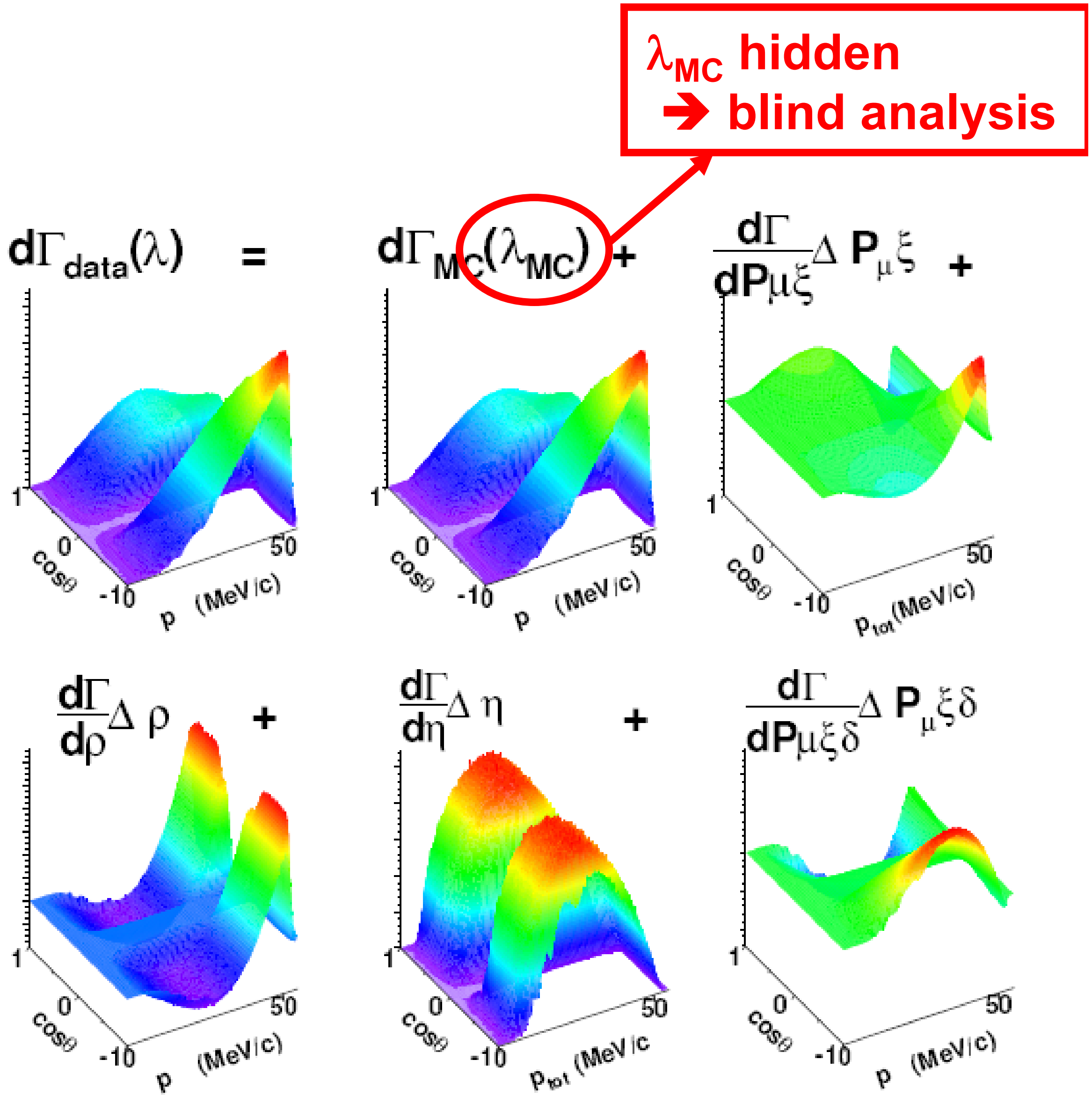
Acceptance of the **TWIST** spectrometer

Fitting the data distributions

- Decay distribution is linear in ρ, η, $P_\mu\xi$, and $P_\mu\xi\delta$, so a fit to first order expansion is exact.

- Fit data to simulated (MC) base distribution with *hidden assumed parameters*,

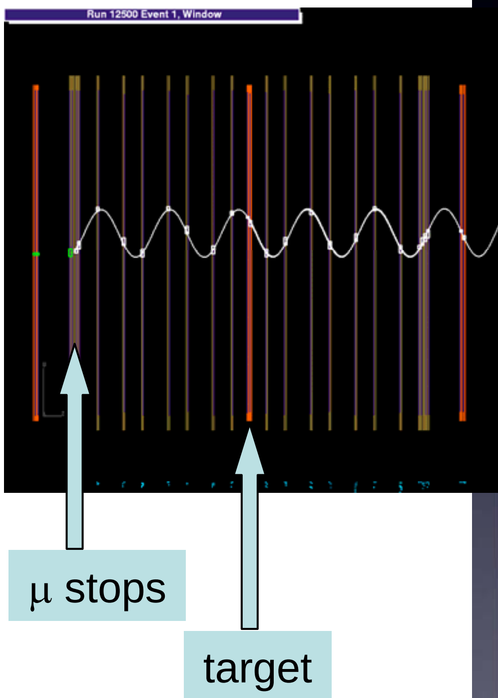
  $\lambda_{MC} = (\rho, \eta, P_\mu\xi_{|P_\mu\xi\delta}, P_\mu\xi\delta)$ plus MC-generated distributions from analytic derivatives, times fitting parameters ($\Delta\lambda$) representing deviations from base MC. (η is now fixed to global analysis value)

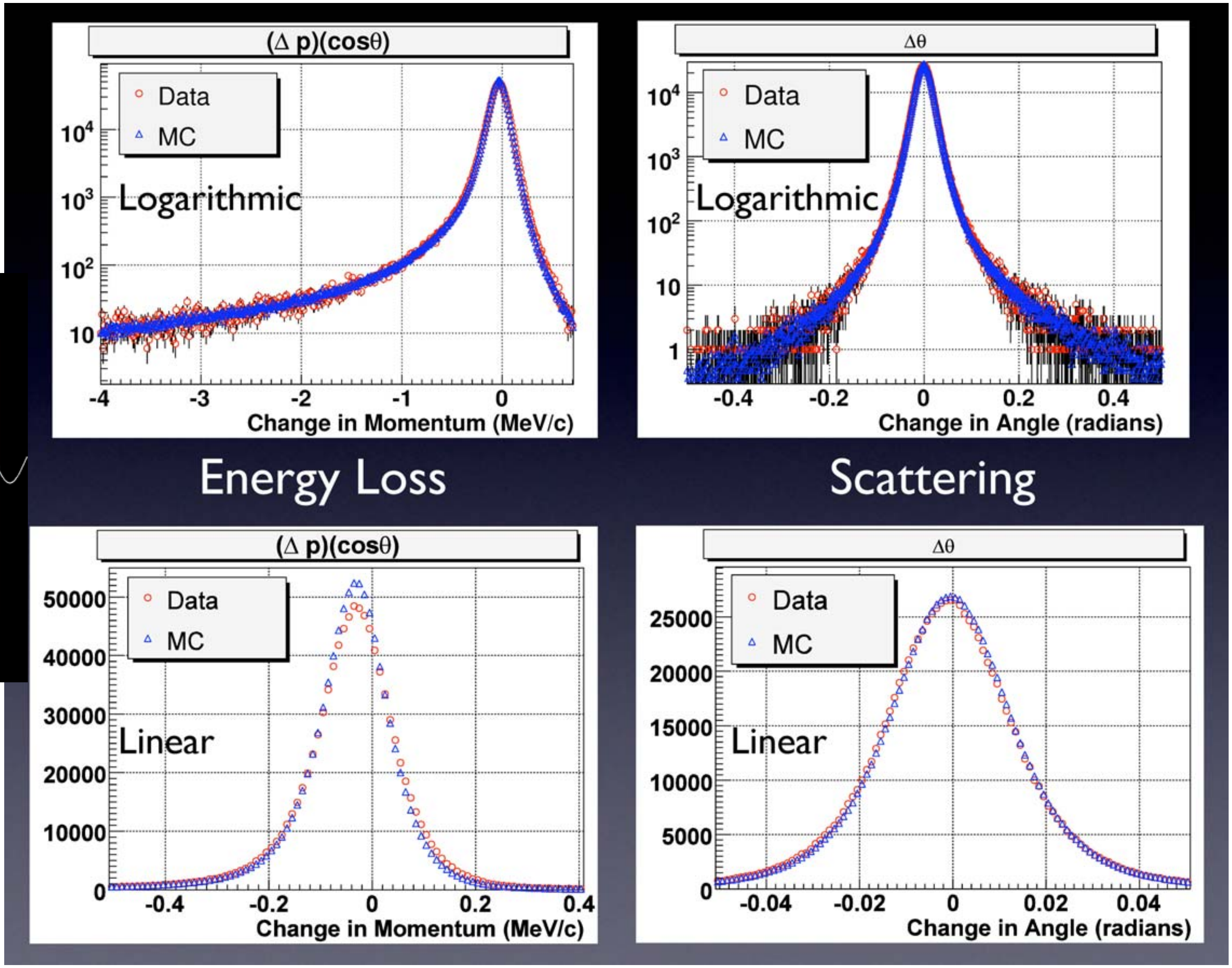


(graphic thanks to Blair Jamieson)

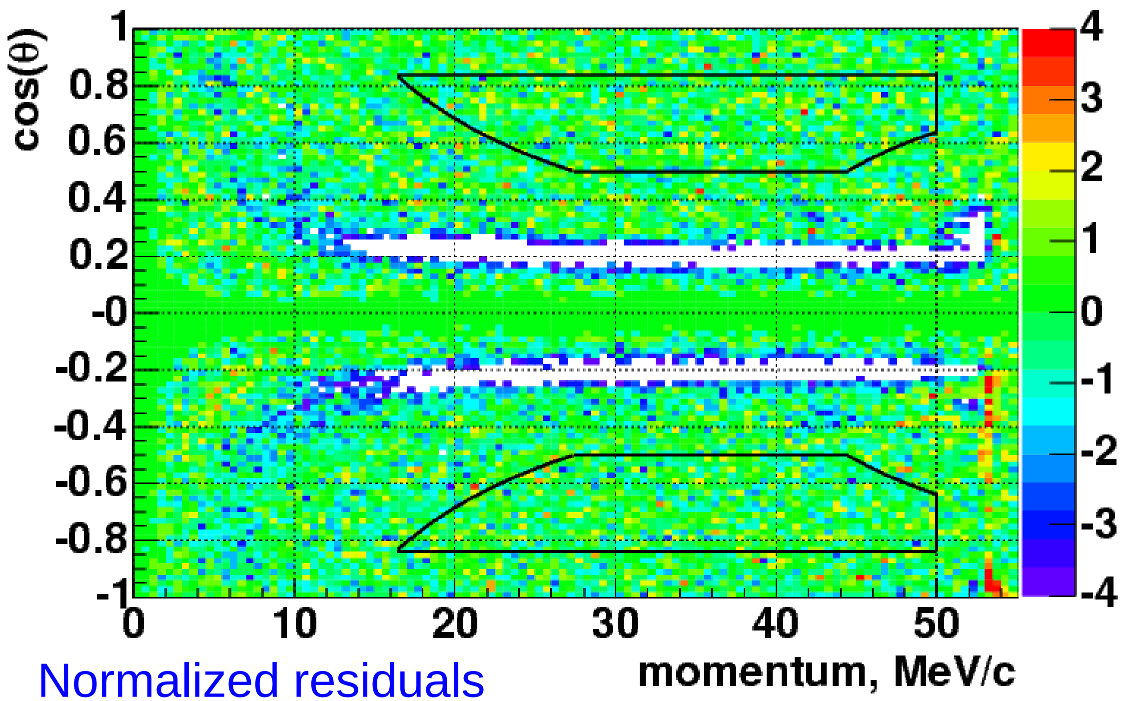
Validating the Monte Carlo with “upstream stops”

(DS-Fit)-(US-Fit)



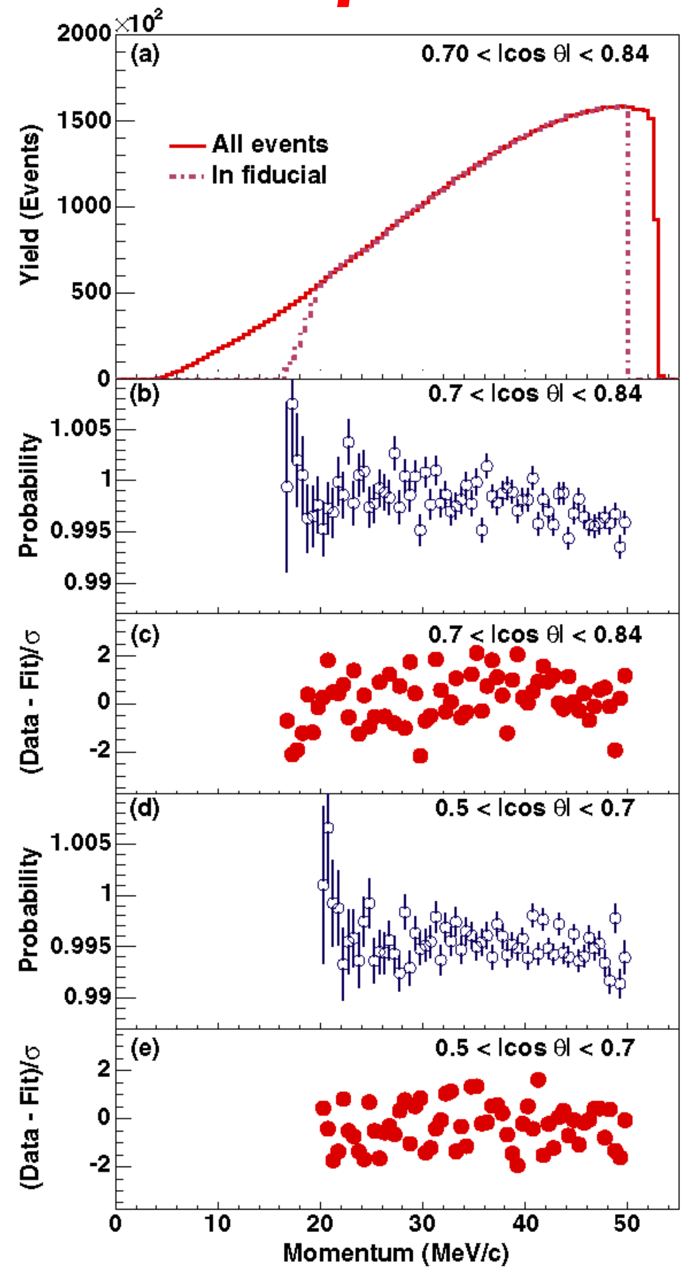


Fitting the 2002 data to determine ρ and δ



Normalized residuals
[(Data-Fit)/sigma] of the 2-d momentum-angle fit

Fit describes the data well, even when
extrapolated far outside the fiducial region



Angle-integrated results

First *TWIST* results for ρ and δ

- From Fall, 2002 run:
  - ρ = 0.75080 ± 0.00032 (stat) ± 0.00097 (syst) ± 0.00023 (η)

    J. Musser et al., PRL **94**, 101805

  - δ = 0.74964 ± 0.00066 (stat) ± 0.00112 (syst)

    A. Gaponenko et al., PRD **71**, 071101

Systematics in the first measurements

TABLE II. Contributions to the systematic uncertainty in ρ. Average values are given for those denoted (av), which are considered set dependent when performing the weighted average of the data sets.

Effect	Uncertainty
Chamber response (av)	$\pm 0.000\,51$
Stopping target thickness	$\pm 0.000\,49$
Positron interactions	$\pm 0.000\,46$
Spectrometer alignment	$\pm 0.000\,22$
Momentum calibration (av)	$\pm 0.000\,20$
Theoretical radiative corrections [12]	$\pm 0.000\,20$
Track selection algorithm	$\pm 0.000\,11$
Muon beam stability (av)	$\pm 0.000\,04$
Total in quadrature	$\pm 0.000\,93$
Scaled total	$\pm 0.000\,97$

Systematic uncertainties typically determined from data sets with a possible problem exaggerated or by MC done with an exaggerated 'defect' put into detector

TABLE II. Contributions to the systematic uncertainty for δ. Average values are denoted by (ave), which are considered set-dependent when performing the weighted average of data sets.

Effect	Uncertainty
Spectrometer alignment	$\pm 0.00\,061$
Chamber response(ave)	$\pm 0.00\,056$
Positron interactions	$\pm 0.00\,055$
Stopping target thickness	$\pm 0.00\,037$
Momentum calibration(ave)	$\pm 0.00\,029$
Muon beam stability(ave)	$\pm 0.00\,010$
Theoretical radiative corrections[9]	$\pm 0.00\,010$
Upstream/downstream efficiencies	$\pm 0.00\,004$

The same effects tend to dominate the systematic uncertainties for both ρ and δ

Global Analysis

Use general form of interaction:

$$M = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=R,L}} g^{\gamma}_{\varepsilon\mu} \left\langle \bar{e}_{\varepsilon} \,|\, \Gamma^{\gamma} \,|\, (\nu_e)_n \right\rangle \left\langle (\bar{\nu}_{\mu})_m \,|\, \Gamma_{\gamma} \,|\, \mu_{\mu} \right\rangle$$

- Follow Fetscher, Gerber, Johnson formulation (Phys. Lett. **173B**, 102 (1986))

Global Analysis

$$Q_{RR} = \frac{1}{4}|g^S_{RR}|^2 + |g^V_{RR}|^2,$$

$$Q_{LR} = \frac{1}{4}|g^S_{LR}|^2 + |g^V_{LR}|^2 + 3|g^T_{LR}|^2,$$

$$Q_{RL} = \frac{1}{4}|g^S_{RL}|^2 + |g^V_{RL}|^2 + 3|g^T_{RL}|^2,$$

$$Q_{LL} = \frac{1}{4}|g^S_{LL}|^2 + |g^V_{LL}|^2,$$

$$B_{LR} = \frac{1}{16}|g^S_{LR} + 6g^T_{LR}|^2 + |g^V_{LR}|^2,$$

$$B_{RL} = \frac{1}{16}|g^S_{RL} + 6g^T_{RL}|^2 + |g^V_{RL}|^2,$$

$$I_\alpha = \frac{1}{4}[g^V_{LR}(g^S_{RL} + 6g^T_{RL})^* + (g^V_{RL})^*(g^S_{LR} + 6g^T_{LR})]$$
$$= (\alpha + i\alpha')/2A,$$

$$I_\beta = \frac{1}{2}[g^V_{LL}(g^S_{RR})^* + (g^V_{RR})^* g^S_{LL}] = -2(\beta + i\beta')/A$$

Constraints:

$$0 \le Q_{\epsilon\mu} \le 1, \quad \text{where } \epsilon, \mu = R, L,$$

$$0 \le B_{\epsilon\mu} \le Q_{\epsilon\mu}, \quad \text{where } \epsilon\mu = RL, LR,$$

$$|I_\alpha|^2 \le B_{LR}B_{RL}, \qquad |I_\beta|^2 \le Q_{LL}Q_{RR},$$

Normalization:

$$Q_{RR} + Q_{LR} + Q_{RL} + Q_{LL} = 1$$

Note that $Q_{LL} \approx 1$

Global Analysis

Relation to muon decay observables:

$$\rho = \frac{3}{4} + \frac{1}{4}(Q_{LR} + Q_{RL}) - (B_{LR} + B_{RL}),$$

$$\xi = 1 - 2Q_{RR} - \frac{10}{3}Q_{LR} + \frac{4}{3}Q_{RL} + \frac{16}{3}(B_{LR} - B_{RL}),$$

$$\xi\delta = \frac{3}{4} - \frac{3}{2}Q_{RR} - \frac{7}{4}Q_{LR} + \frac{1}{4}Q_{RL} + (B_{LR} - B_{RL}),$$

e^+_L {

$$\xi' = 1 - 2Q_{RR} - 2Q_{RL},$$

$$\xi'' = 1 - \frac{10}{3}(Q_{LR} + Q_{RL}) + \frac{16}{3}(B_{LR} + B_{RL}),$$

rad. decay { $\bar{\eta} = \frac{1}{3}(Q_{LR} + Q_{RL}) + \frac{2}{3}(B_{LR} + B_{RL}),$

e^+_T { $\eta = (\alpha - 2\beta)/A, \qquad \eta'' = (3\alpha + 2\beta)/A.$

Global Analysis

2005 Input:

Parameter	Value
ρ	0.7518 ± 0.0026
	$0.75\,080 \pm 0.00\,105$ [a]
δ	0.7486 ± 0.0038
	$0.74\,964 \pm 0.00\,130$
$P_\mu \xi$	1.0027 ± 0.0085 [b]
$P_\mu \xi \delta / \rho$	$0.99\,787 \pm 0.00\,082$ [b]
ξ'	1.00 ± 0.04
ξ''	0.65 ± 0.36
$\bar{\eta}$	0.02 ± 0.08
α/A	0.015 ± 0.052 [c]
β/A	0.002 ± 0.018 [c]
η	0.071 ± 0.037 [d]
η''	0.105 ± 0.052 [d]
α'/A	-0.047 ± 0.052 [e]
	-0.0034 ± 0.0219 [f]
β'/A	0.017 ± 0.018 [e]
	-0.0005 ± 0.0080 [f]

2005 Output:

Parameter	Fit Result ($\times 10^3$)
Q_{RR}	$<1.14(0.60 \pm 0.38)$
Q_{LR}	$<1.94(1.22 \pm 0.53)$
B_{LR}	$<1.27(0.72 \pm 0.40)$
Q_{RL}	$<44(26 \pm 13)$
B_{RL}	$<10.9(6.4 \pm 3.3)$
Q_{LL}	$>955(973 \pm 13)$
α/A	0.3 ± 2.1
β/A	2.0 ± 3.1
α'/A	-0.1 ± 2.2
β'/A	-0.8 ± 3.2

Reducing the leading systematics

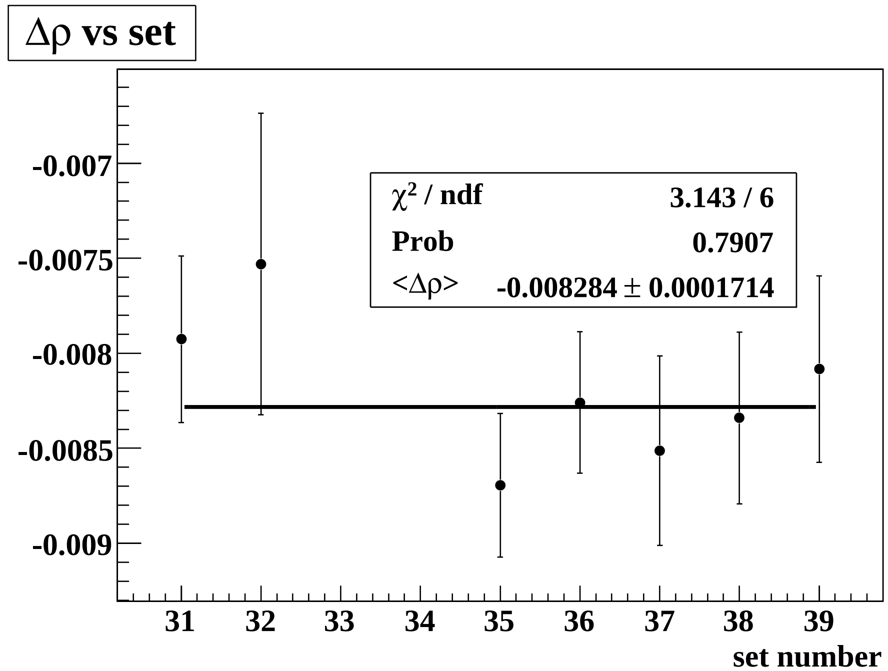
- Issues that were unique to 2002 data
  - Stopping target thickness uncertainty
  - Chamber orientation uncertainty with respect to magnetic field
- Improvements in 2004 data
  - Chamber response
    - Improved gas system regulation and monitoring
    - Improved determination of foil geometry
    - Improved treatment of drift chamber behavior
  - Positron interactions better understood
  - Detector fully instrumented
  - Improved alignment techniques and understanding of uncertainties
  - New momentum calibration techniques (uncertainty is statistical)
  - Radiative corrections uncertainty evaluated

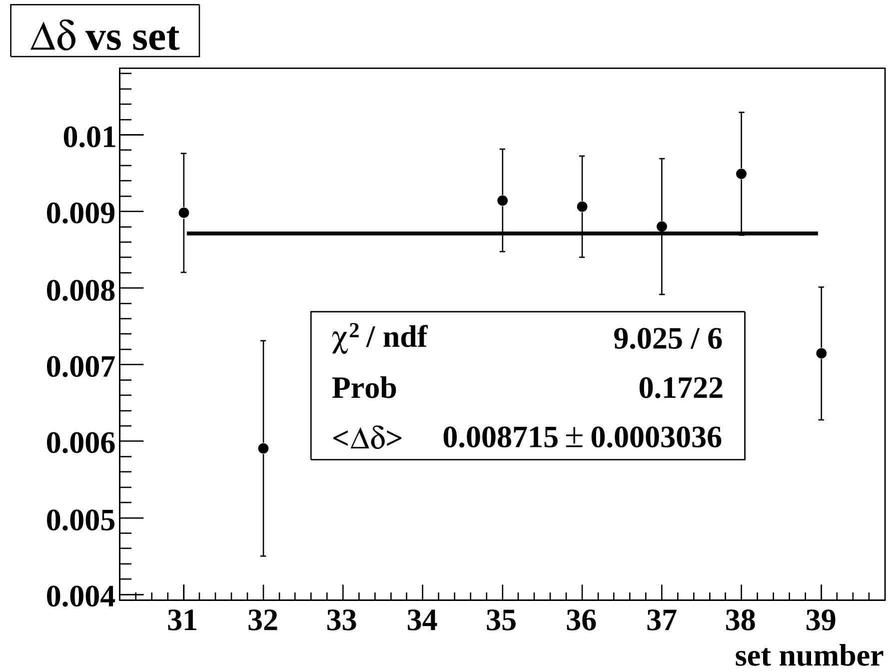
Systematic uncertainties for 2004 data: ρ and δ

Systematic uncertainties	ρ (×10⁴)		δ (×10⁴)	
	2002	2004	2002	2004
Chamber response (ave)	5.1	2.9	6.1	5.2
Stopping target thickness	4.9	<0.1	3.7	<0.1
Positron interactions	4.6	1.6	5.5	0.9
Spectrometer alignment	2.2	0.3	6.1	0.3
Momentum calibration (ave)	2.0	2.9	2.9	4.1
Theoretical radiative correction	2.0	<0.1	1.0	<0.1
Other	1.2	1.1	1.1	0.4
Total in quadrature	**9.2**	**4.6**	**11.3**	**6.7**

Consistency Checks: ρ and δ

- Data sets for 2004 analysis
- Δ's from fits to MC
- No corrections applied
- Decay parameters in BB still hidden

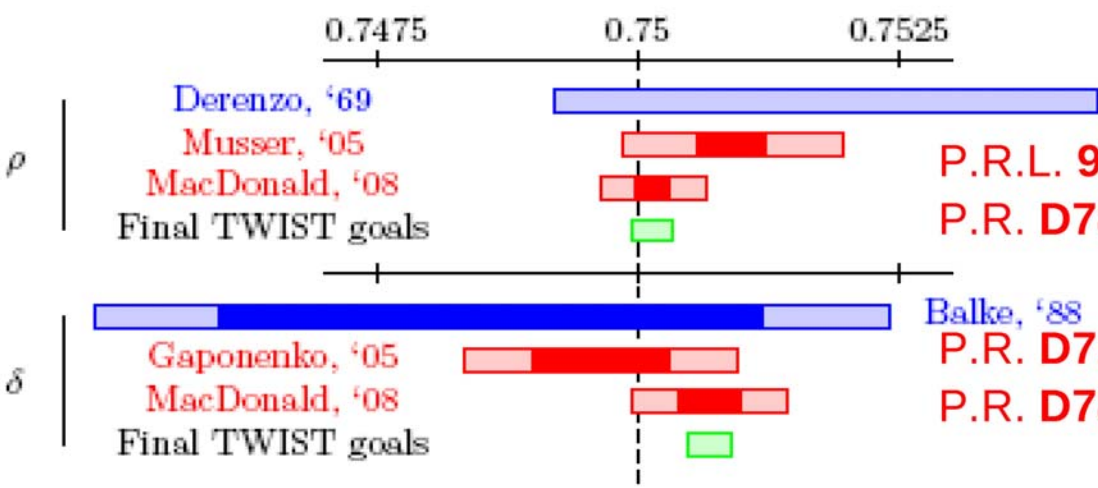




Results to date

- From Fall, 2002 run:
  - ρ = 0.75080 ± 0.00032 (stat) ± 0.00097 (syst) ± 0.00023 (η)
  - δ = 0.74964 ± 0.00066 (stat) ± 0.00112 (syst)
- From Fall, 2004 run:
  - ρ = 0.75014 ± 0.00017 (stat) ± 0.00044 (syst) ± 0.00011 (η)
  - δ = 0.74964 ± 0.00030 (stat) ± 0.00067 (syst)

R. McDonald et al., PRD **78**, 032010



P.R.L. **94**, 101805(2005)

P.R. **D78**, 032010(2008)

P.R. **D71**, 071171**R**(2005)

P.R. **D78**, 032010(2008)

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Global Analysis Results

	Pre-*TWIST*	2002 Data	2004 Data
$\|g^S_{LR}\|$	<0.125	<0.088	<0.074
$\|g^V_{LR}\|$	<0.066	<0.036	<0.025
$\|g^T_{LR}\|$	<0.036	<0.025	<0.021
Q_R^μ	<0.0051	<0.0031	<0.0024

90% confidence limits

Final Uncertainty Goals

	Published		Final (est.)	
	Statistics	Systematics	Statistics	Systematics
ρ	1.7	4.4	1.3	2.4
δ	3.0	6.7	2.3	3.2

all values in units of 10^{-4}

Final Publications in 2009

TWIST Collaboration

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