TWIST: precision measurement of the muon decay parameters

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Abstract. The TRIUMF Weak Interaction Symmetry Test (TWIST) collaboration has nearly completed the world's most precise measurement of the energy-angle spectrum of positrons from the decay of highly polarized muons. A simultaneous measurement of the muon decay parameters ρ, δ, and $\mathcal{P}_\mu^\pi \xi$ tests the Standard Model (SM) in a purely leptonic process and provides improved limits for relevant extensions to the SM. Since the completion of data taking in 2007, the analysis has focused on reducing systematic uncertainties, estimating residual biases, and evaluating consistency checks. The analysis was blind with respect to the central values of the parameters, so the results were unknown until the values of the hidden parameters were revealed. The total uncertainties for the parameters are dominated by systematic uncertainties. While there are still some subtle systematic effects that preclude final values of the decay parameters, the goal of an order-of-magnitude improvement on pre-TWIST precisions appears to have been achieved.

1. Introduction

Muon decay is an excellent laboratory for testing the electroweak Standard Model (SM). It is a purely leptonic process with the positive muon decaying into a positron and two neutrinos. The matrix element for the most general Lorentz invariant, derivative free expression [1] is described by 10 complex model-independent couplings g_{em}^k:

$$M = \frac{4G_F}{\sqrt{2}} \sum_{\substack{e=L,R \\ m=L,R \\ \kappa=S,V,T}} g_{ij}^\kappa \langle \bar{\psi}_{e_e} | \Gamma^\kappa | \psi_{\nu_e} \rangle \langle \bar{\psi}_{\nu_\mu} | \Gamma_\kappa | \psi_{\mu_m} \rangle, \tag{1}$$

where L and R are left and right handed leptons, and S, V, and T are scalar, vector, and tensor iteractions. In the SM $g_{LL}^V = 1$ and $g_{em}^k = 0$ otherwise. Experimentally only the positron is measured and the decay spectrum is usually written in terms of four parameters[2, 3, 4]: the

Michel parameter ρ, δ, $\mathcal{P}_\mu^\pi \xi$, and η. The differential decay rate is then

$$\frac{d^2\Gamma}{dx\ d\cos\theta} = \frac{1}{4}m_\mu W_{\mu e}^4 G_F^2 \sqrt{x^2 - x_0^2} \cdot \{\mathcal{F}_{IS}(x, \rho, \eta) + \mathcal{P}_\mu \cos\theta \cdot \mathcal{F}_{AS}(x, \xi, \delta)\}$$

with

$$W_{\mu e} = \frac{m_\mu^2 + m_e^2}{2m_\mu}, \ x = \frac{E_e}{W_{\mu e}}, \ x_0 = \frac{m_e}{W_{\mu e}}, \ \mathcal{P}_\mu = |\vec{\mathcal{P}}_\mu|, \ \cos\theta = \frac{\vec{\mathcal{P}}_\mu \cdot \vec{p}_e}{|\vec{\mathcal{P}}_\mu|\,|\vec{p}_e|}$$

$$\mathcal{F}_{IS}(x, \rho, \eta) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$

$$\mathcal{F}_{AS}(x, \xi, \delta) = \frac{1}{3}\sqrt{x^2 - x_0^2}\xi \left[1 - x + \frac{2}{3}\delta\left\{4x - 3 + \left(\sqrt{1 - x_0^2} - 1\right)\right\}\right]$$

The neutrino mass is neglected. Radiative corrections are not explicitly shown, but are significant and must be evaluated within the SM to a precision compatible with the experiment. The isotropic term $\mathcal{F}_{IS}(x)$ depends on the decay parameters ρ and η, while the asymmetric part $\mathcal{F}_{AS}(x)$ depends on δ and ξ. The asymmetric part is multiplied by the polarization of the muon at the time of decay, $\mathcal{P}_\mu$, which may evolve over the 2.2 μs mean lifetime of the muon from the polarization $\mathcal{P}_\mu^\pi$ at the time of the muon's birth, e.g., in pion decay at rest. The decay parameters can be written as bilinear combinations of the g_{em}^k. The SM predictions are $\rho = \delta = 3/4$, $\mathcal{P}_\mu^\pi = \xi = 1$, and $\eta = 0$. Precision measurements of these parameters will test the SM predictions and are sensitive to extensions to the SM. The TRIUMF Weak Interaction Symmetry Test (TWIST) experiment has made new measurements of three of these parameters resulting in improved constraints on extensions to the SM.

Prior to the TWIST experiment, the three decay parameters were known with uncertainties in the range of 3.5-8.5 parts per thousand. Intermediate TWIST results have already reduced those uncertainties to 0.7-3.8 parts per thousand.[5, 6] The decay parameters measured by TWIST contribute to a larger set derived from other observables that can be analyzed in terms of the generalized matrix element in Eq. 1. A global analysis[5, 8] reveals consistency with the SM, where the vector coupling for muons and electrons of left-handed chirality is the only non-zero term. The results from TWIST restrict the upper limits of other terms, especially those for left-handed electrons and right-handed muons. For example, the probability for right-handed muon couplings in muon decay was reduced by a factor of two to less than 2.4×10^{-3} (90% confidence).

2. Experimental details

The TWIST experiment used a 500 MeV proton beam incident on a graphite production target. The highly polarized positive muons from pions that decayed on the surface of the target were transported by the M13 surface muon channel to the detector, which was located in the bore of a 2 T solenoidal magnet. The muons were guided into the superconducting solenoidal field along its symmetry axis to enter a high-precision, low-mass stack of proportional and drift chambers.[9] The TWIST spectrometer is shown in Fig. 1. The muons were ranged to stop predominantly in a high-purity metal foil at the center of the symmetric stack. Data sets were taken with two foil stopping targets, silver (thickness 30.9 μm) and aluminum (thickness 71.6 μm). Tracks from decay positrons were sampled by the low-mass drift chambers in a helium gas environment. The data-taking phase of the experiment was completed in 2007. Analysis provides two-dimensional distributions of positron angle and momentum (or energy) whose shape depends on the decay parameters. With a muon rate of order 2–5$\times 10^3$ s^{-1}, data sets of 10^9 events could be obtained in a few days. Much care is taken to test for and avoid the introduction of any bias. The
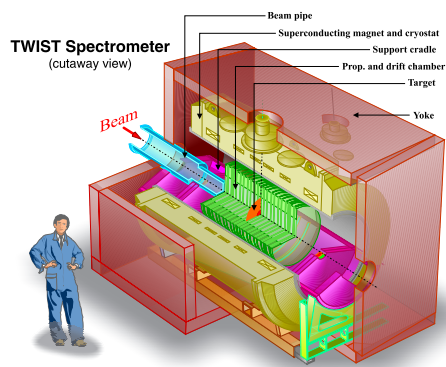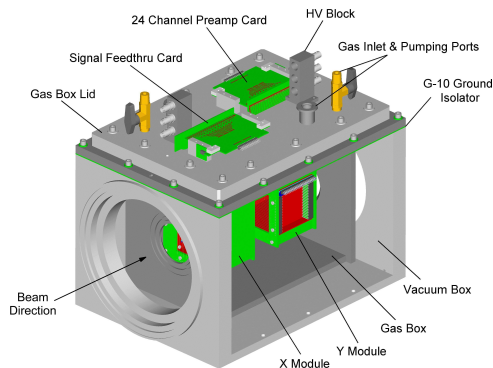
Figure 1. The TWIST spectrometer.

Figure 2. The TWIST TECs.

fiducial cuts are symmetric for upstream and downstream decays, and are selected to maximize sensitivity to the decay parameters while reducing systematic uncertainties.

To determine the incoming muon beam characteristics (size, position, divergence, and correlations), a beam monitor detector was inserted to measure the beam before it entered the solenoid (see Fig. 2). This detector is a pair of time expansion chambers (TECs) recording the position and angle of each incident muon.[10] Because they caused multiple scattering and hence muon depolarization, the TECs were typically removed for precise decay measurements, but the beam characteristics measured between data sets formed an essential input to the simulation and analysis of decay data.

3. Analysis procedures

The important principle of TWIST analysis is the comparison of energy-angle two-dimensional distributions of data to similar ones derived from a GEANT3 simulation. Both are subjected to essentially the same analysis, allowing bias and inefficiencies to be included in an equivalent way to reduce the dependence of the result on the specific analysis procedure. This places great importance on the accuracy and detail of the simulation, which includes not only standard physics processes but also a detailed description of the beam, magnetic field, geometry, and detector response. Decay parameters are obtained by a "blind" fit of the two-dimensional data distribution to that of a base distribution of simulated events, generated with hidden muon decay parameters, plus distributions corresponding to the two-dimensional spectrum shape of first derivatives of the spectrum with respect to decay parameters (or combinations) ρ, ξ, and $\xi\delta$, also derived from simulated events.

4. Systematic uncertainties

The procedure of fitting the difference of two spectra in terms of derivatives also plays a key role in evaluation of systematic uncertainties, by finding the effect on the decay parameters when an identified source of systematic uncertainty is changed (often by an exaggerated amount) in one of the spectra. This is most commonly achieved with two simulated spectra. The systematic contributions as determined prior to revealing the hidden parameters of the blind analysis are listed, along with statistical uncertainties, in Table 1.

Four sources dominate for ρ and δ: positron interactions, momentum calibration, chamber response, and external uncertainties due to radiative corrections and the assumed value of η. The first three were improved substantially compared to our intermediate results.[5] The positron interactions systematic relates the possible inaccuracy in our simulation of reproducing positron energy loss in the stopping target and detector elements, due primarily to bremsstrahlung,

Table 1. Systematic and statistical uncertainties for ρ, δ, and $\mathcal{P}_\mu^\pi\xi$ prior to revealing hidden parameters.

Uncertainties	$\rho(\times10^{-4})$	$\delta(\times10^{-4})$	$\mathcal{P}_\mu^\pi\xi(\times10^{-4})$
Positron interactions	1.8	1.6	0.7
Momentum calibration	1.2	1.2	1.5
Chamber response	1.0	1.8	2.3
External uncertainties	1.3	0.6	1.2
Resolution	0.6	0.7	1.5
Spectrometer alignment	0.2	0.3	0.2
Beam stability	0.2	0.0	0.3
Depolarization in fringe field			+15.8, -4.0
Depolarization in stopping material			3.2
Background muons			1.0
Depolarization in production target			0.3
Total systematics in quadrature	2.8	2.9	+16.5, -6.3
Statistical uncertainty	0.9	1.6	3.5
Total uncertainty	3.0	3.3	+16.9, -7.2

delta-ray production, and ionization. It was better constrained by comparisons of identified interactions observed in the data and in the simulation. Chamber response refers to the conversion of drift chamber time information to spatial information used in helix fitting and evaluation of the momentum and angle of each track. It was improved by more precise monitoring and control of atmospheric influences that could change chamber cell geometry. In addition, a method was developed[11] by which the detector space-time relations (STRs) were modified for each plane to minimize positron decay track fit residuals. The tracking bias was reduced by applying the procedure also to simulations. Changes to the spatial isochrone shapes varied from zero to $\sim40\ \mu$m in drift cells of 4×4 mm^2. The maximum positron energy provides a calibration feature that is used to reduce the energy scale systematic. Since energy loss depends on the track angle primarily with a dependence on $1/(\cos\theta)$ due to the planar geometry of the detector, the energy region near the kinematic endpoint of 52.8 MeV/c is matched for data and simulation for small bins of $\cos\theta$. The data-simulation relative energy calibration procedure has undergone improvements to become more robust to fitting conditions.

The asymmetry parameter ξ is also subject to uncertainties from these sources, but they are overshadowed by other unique uncertainties related to depolarization, as shown in Table 1. Depolarization in the fringe field and in the muon stopping target result in $\mathcal{P}_\mu < \mathcal{P}_\mu^\pi$, comprising the largest contributions to systematic uncertainties for $\mathcal{P}_\mu^\pi\xi$. They were also considerably improved for this analysis compared to the intermediate result.[6] Fringe field depolarization systematics depend on the accuracy with which the muon spin evolution can be simulated as the beam passes through significant radial field components at the solenoid entrance. The simulation in turn depends on two ingredients: an accurate field map, and precise knowledge of the position and direction of the muons in the beam. Depolarization in the stopping target from muon spin relaxation as the spins interact with the target material is assessed from the measured time dependence of the asymmetry.

5. Results
Fourteen data sets were used to extract decay parameters ρ and δ, seven with each of the Ag and Al targets. Only nine sets were used for $\mathcal{P}_\mu^\pi\xi$; the other five were deemed to have poorly
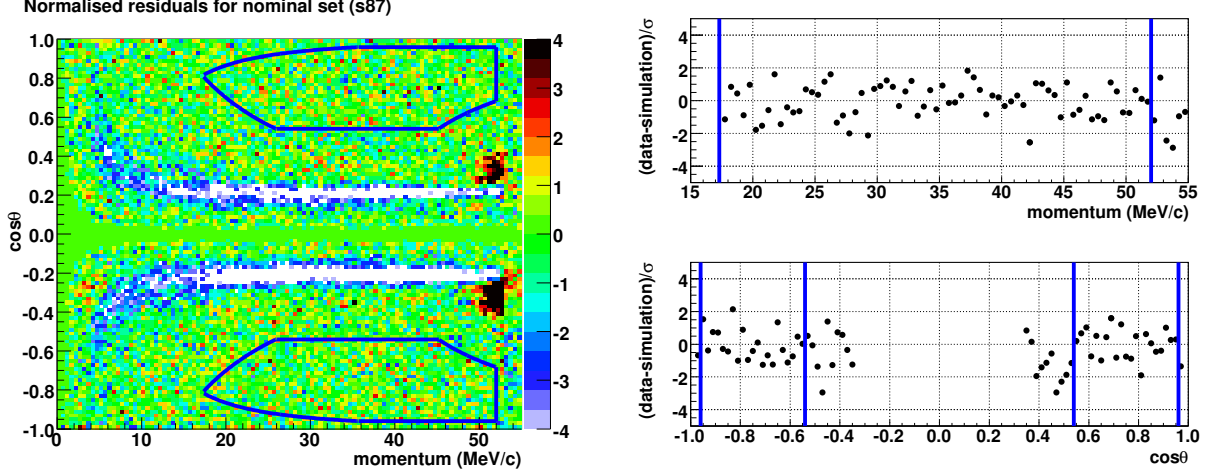
Figure 3. Comparison of data with a fit to simulation in terms of normalized residuals for one data set. The left plot is the entire range in momentum and $\cos\theta$, while the right plot shows projections of data within the fiducial (indicated by the solid line) onto the two axes.

controlled polarization systematic uncertainties. The fringe field was not well measured for the sets at 1.96 T and 2.04 T field, there was added multiple scattering and thus increased fringe field depolarization for the set with the TECs in place, while two other sets were intentionally mis-steered to evaluate depolarization systematics in the fringe field. Fits to constant means for the difference values give reduced χ^2 values of 14.0/13 (ρ), 17.7/13 (δ), and 9.7/8 ($\mathcal{P}_\mu^\pi \xi$) respectively.

A visualization of the fit of one data set (set 87) in terms of residuals is shown in Fig. 3. It also shows an outline of the range of $(p, \cos\theta)$ used to determine the decay parameters. The limits of this fiducial range in momentum (total, transverse, and longitudinal) and angle are those within which the systematic biases and uncertainties are considered to be well controlled. For all fourteen data sets, there were 11×10^9 events, of which 0.55×10^9 passed event selection criteria and were within this fiducial range. Simulation data sets were about 2.7 times larger on average.

After revealing the hidden parameters, the results for the three decay parameters were:

$$
\begin{aligned}
\rho &= 0.74991 \pm 0.00009(\text{stat}) \pm 0.00028(\text{syst}) \\
\delta &= 0.75072 \pm 0.00016(\text{stat}) \pm 0.00029(\text{syst}) \\
\mathcal{P}_\mu^\pi \xi &= 1.00084 \pm 0.00035(\text{stat}) \, {}^{+0.00165}_{-0.00063}(\text{syst})
\end{aligned}
$$

These values are consistent with the SM predictions of 0.75, 0.75, and 1.0. The results are compared graphically to prior published results in Fig. 4. Also plotted is the product $\mathcal{P}_\mu^\pi \xi \delta / \rho = 1.00192 \, {}^{+0.00167}_{-0.00066}$ (with correlations taken into account). This product defines the asymmetry between $\cos\theta = \pm 1$ at the maximum decay positron energy, which is 1.0 in the SM. While the generalized matrix element treatment of Fetscher et al.[1] does not constrain the sign of deviations from the SM values for ρ, δ, and ξ, the product is constrained to be not greater than 1.0. This apparent contradiction has initiated an ongoing reconsideration of potential systematic effects that might have been overlooked in the blind analysis. While no credible cause has yet been identified, we believe the resolution will be in terms of systematic effects. Thus we prefer not to consider the results of the blind analysis as our final physics results, pending the outcome of a more complete re-assessment of potential sources of such effects.
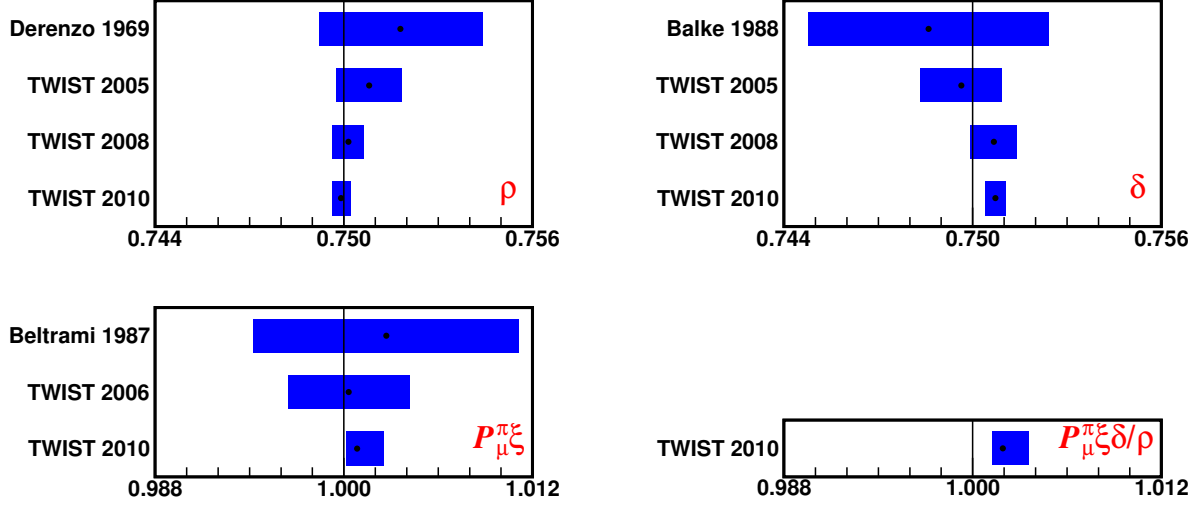
Figure 4. Summary of previously published values with uncertainties added in quadrature for three muon decay parameters, including those prior to TWIST, along with the results of this blind analysis. The combination $\mathcal{P}_\mu^\pi \xi \delta / \rho$ is also shown.
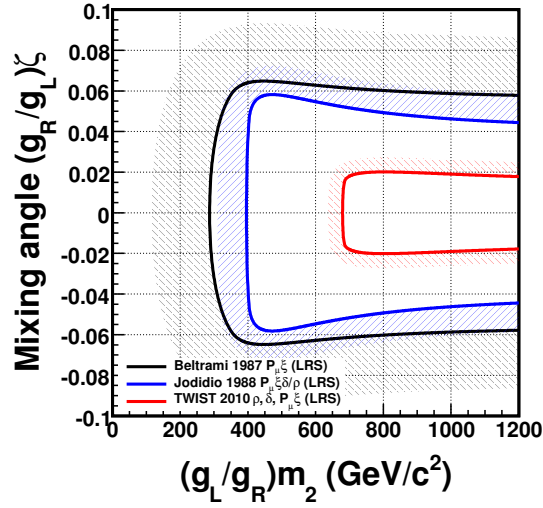

Figure 5. Allowed region of mixing angle and heavy W mass for the general LRS model

Left-right symmetric models extend the SM with a right-handed W.[12] In the generaized model no assumptions are made about the ratio of left-handed to right-handed couplings. The TWIST result for ρ provides the best constraint on the mixing angle between the light and heavy mass eigenstates: W_1 and W_2. The current limit is $|\zeta_g| < 0.020$ (90% C.L.), compared to the pre-TWIST limit of $|\zeta_g| < 0.066$. The lower limit on the mass of W_2 (m_2) comes from the value of $\mathcal{P}_\mu^\pi \xi$ and has been increased from $400\,GeV/c^2$ to $684\,GeV/c^2$. Coupled constraints on the mass for $(g_L/g_R)m_2$ and the mixing angle are shown in Fig. 5.

6. Summary

We have achieved a substantial improvement for the final results of TWIST, compared to intermediate results and to prior experiments. Final checks of consistency and continuing re-evaluation of systematic uncertainties are underway, with the goal of understanding an apparent inconsistency in the product $\mathcal{P}_\mu^\pi \xi \delta / \rho$.

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