

Muon Decay Parameters from *TWIST*

Glen Marshall, TRIUMF (for the *TWIST* Collaboration)
Lake Louise Winter Institute, Lake Louise, February 20–26, 2011



TWIST participants, past and present

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Outline

- ▶ Physics of muon decay
- ▶ The muon beam and the *TWIST* spectrometer
- ▶ Analysis strategies
- ▶ Estimation of systematic uncertainties
- ▶ Results for decay parameters ρ , δ , and $\mathcal{P}_\mu \xi$

Decay parameters

- Muon decay parameters $\rho, \eta, \mathcal{P}_\mu \xi, \delta$

- muon differential decay rate vs. energy and angle:

$$\frac{d^2\Gamma}{dx d\cos\theta} = \frac{1}{2\pi^3} m_\mu W_{\mu e}^4 G_F^2 \sqrt{x^2 - x_0^2} \cdot$$

$$\{\mathcal{F}_{IS}(x, \rho, \eta) \pm \mathcal{P}_\mu \cos\theta \cdot \mathcal{F}_{AS}(x, \xi, \delta)\} + R.C.$$

- where

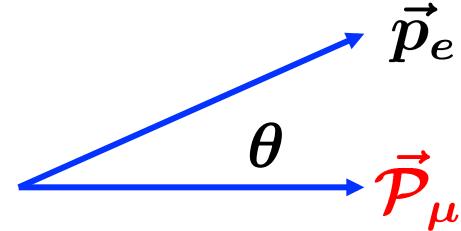
$$\mathcal{F}_{IS}(x, \rho, \eta) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$

$$\mathcal{F}_{AS}(x, \xi, \delta) = \frac{1}{3}\xi\sqrt{x^2 - x_0^2} \left[1 - x + \frac{2}{3}\delta \left\{ 4x - 3 + (\sqrt{1 - x_0^2} - 1) \right\} \right]$$

and $W_{\mu e} = \frac{m_\mu^2 + m_e^2}{2m_\mu}$, $x = \frac{E_e}{W_{\mu e}}$, $x_0 = \frac{m_e}{W_{\mu e}}$.



L. Michel



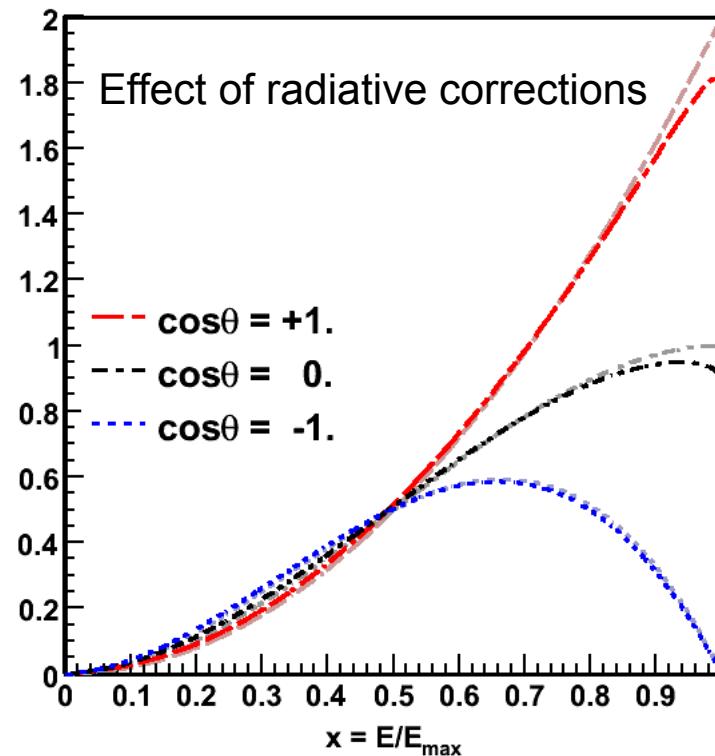
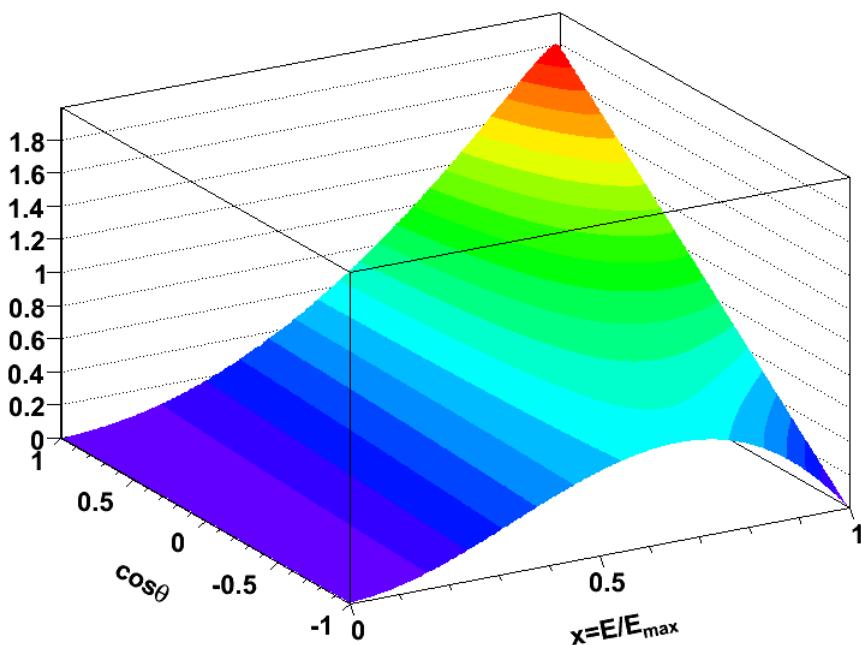
L. Michel, Proc. Phys. Soc. A63, 514 (1950).

C. Bouchiat and L. Michel, Phys. Rev. 106, 170 (1957).

T. Kinoshita and A. Sirlin, Phys. Rev. 107, 593 (1957).

T. Kinoshita and A. Sirlin, Phys. Rev. 108, 844 (1957).

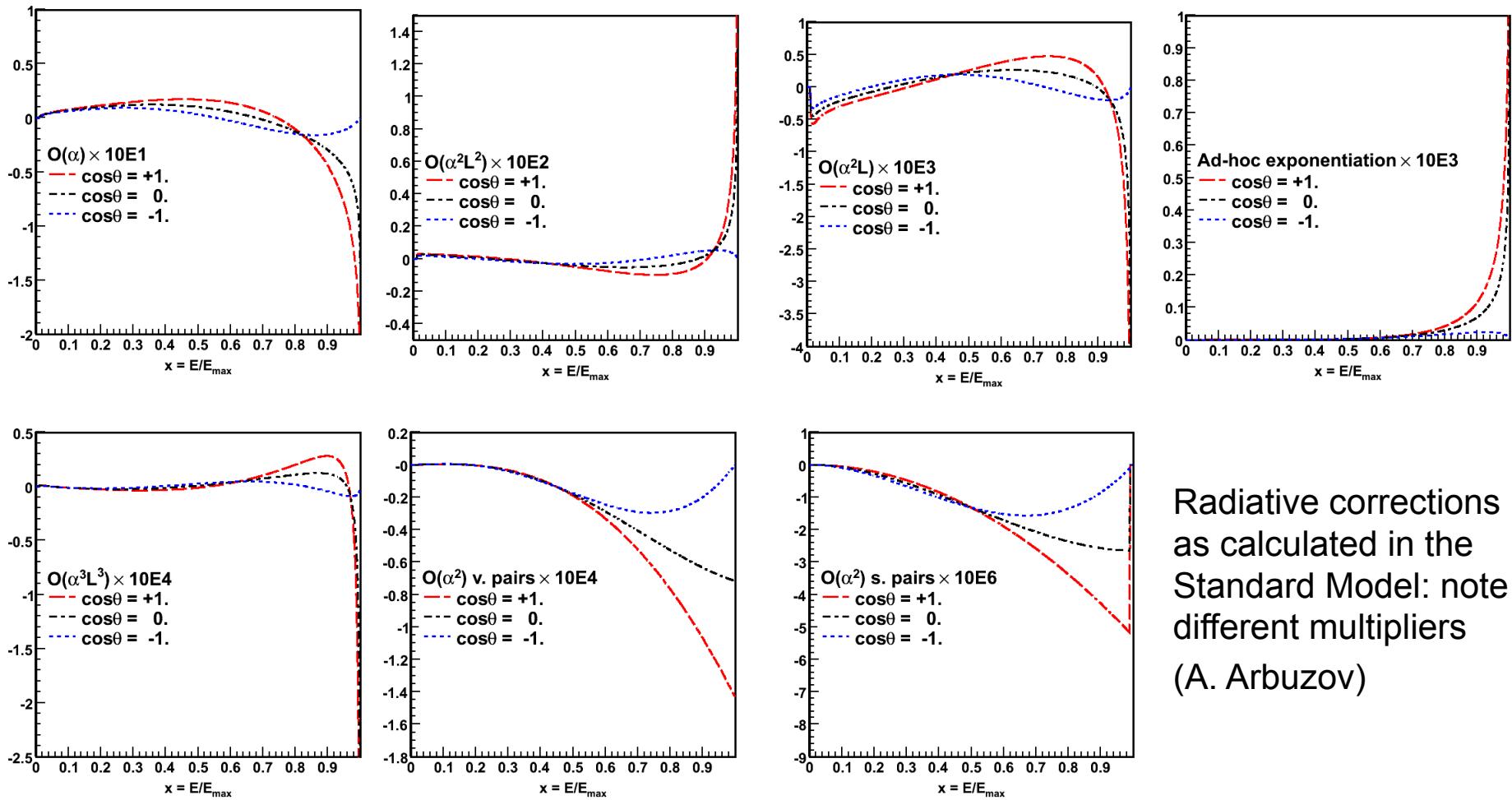
Spectrum shape and radiative corrections



- ▶ Full $\mathcal{O}(\alpha)$ radiative corrections with exact electron mass dependence.
- ▶ Leading and next-to-leading logarithmic terms of $\mathcal{O}(\alpha^2 \mathcal{L}^2)$ and $\mathcal{O}(\alpha^2 \mathcal{L})$, $\mathcal{L} = \ln((m_\mu/m_e)^2)$
- ▶ Leading logarithmic terms of $\mathcal{O}(\alpha^3 \mathcal{L}^3)$.
- ▶ Ignores $\mathcal{O}(\alpha^2 \mathcal{L}^0)$ (2007).

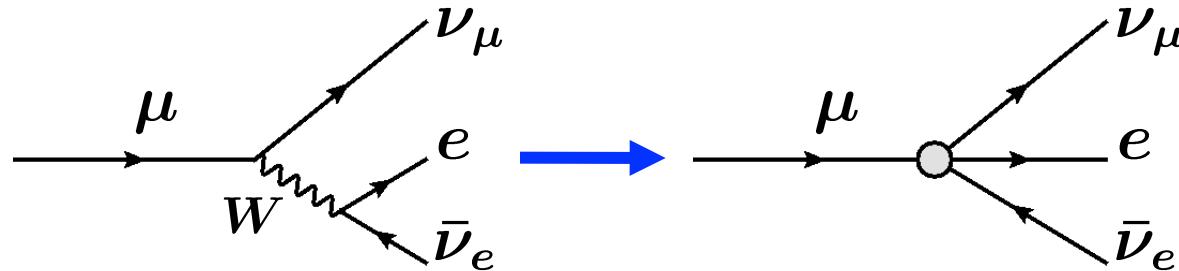
K. Melnikov, J. High Energy Phys. (09):014 (2007)
A. Arbuzov, J. High Energy Phys. (03):063 (2003)
A. Arbuzov et al., Phys. Rev. D66, 93003 (2002)
A. Arbuzov et al., Phys. Rev. D65, 113006 (2002)

Radiative correction contributions



Radiative corrections
as calculated in the
Standard Model: note
different multipliers
(A. Arbuzov)

Matrix elements



$$M = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=R,L}} g_{\varepsilon\mu}^\gamma \langle \bar{e}_\varepsilon | \Gamma^\gamma | (\nu_e)_n \rangle \langle (\bar{\nu}_\mu)_m | \Gamma_\gamma | \mu_\mu \rangle$$

- ▶ General local, Lorentz-invariant, lepton-number conserving interaction.
- ▶ Scalar, vector, tensor ($\Gamma^S, \Gamma^V, \Gamma^T$) interactions, left and right μ, e
- ▶ Decay parameters are bilinear combinations of $g_{\varepsilon\mu}^\gamma$.
- ▶ Probability for decay of μ -handed muon to ε -handed electron:

$$Q_{\varepsilon\mu} = \frac{1}{4} |g_{\varepsilon\mu}^S|^2 + |g_{\varepsilon\mu}^V|^2 + 3(1 - \delta_{\varepsilon\mu}) |g_{\varepsilon\mu}^T|^2$$

- ▶ RH coupling in μ decay in terms of decay parameters:

$$Q_R^\mu = \frac{1}{2} \left[1 + \frac{1}{3} \xi - \frac{16}{9} \xi \delta \right]$$

Fetscher, Gerber and Johnson, Phys. Lett. B173, 102 (1986) .

Pre-TWIST decay parameters

- ▶ From the Review of Particle Physics (SM values)
 - ▶ $\rho = 0.7518 \pm 0.0026$ (0.75)
 - ▶ S.E. Derenzo, Phys. Rev. 184, 1854 (1969) .
 - ▶ $\delta = 0.7486 \pm 0.0026 \pm 0.0028$ (0.75)
 - ▶ B. Balke *et al.*, Phys. Rev. D37, 587 (1988) 587.
 - ▶ $\mathcal{P}_\mu \xi = 1.0027 \pm 0.0079 \pm 0.0030$ (1.00)
 - ▶ I. Beltrami *et al.*, Phys. Lett. B194, 326 (1987).
 - ▶ $\mathcal{P}_\mu (\xi \delta / \rho) > 0.99682$ (90%CL) (1.00)
 - ▶ A. Jodidio *et al.*, Phys. Rev. D34, 1967 (1986), and erratum.
 - ▶ $\eta = 0.011 \pm 0.085$ (0.00)
 - ▶ H. Burkhardt *et al.*, Phys. Lett. 160B, 343 (1985).

The goal of *TWIST* is to find any evidence for new physics that may become apparent by improving the precision of

ρ , δ , and $\mathcal{P}_\mu \xi$

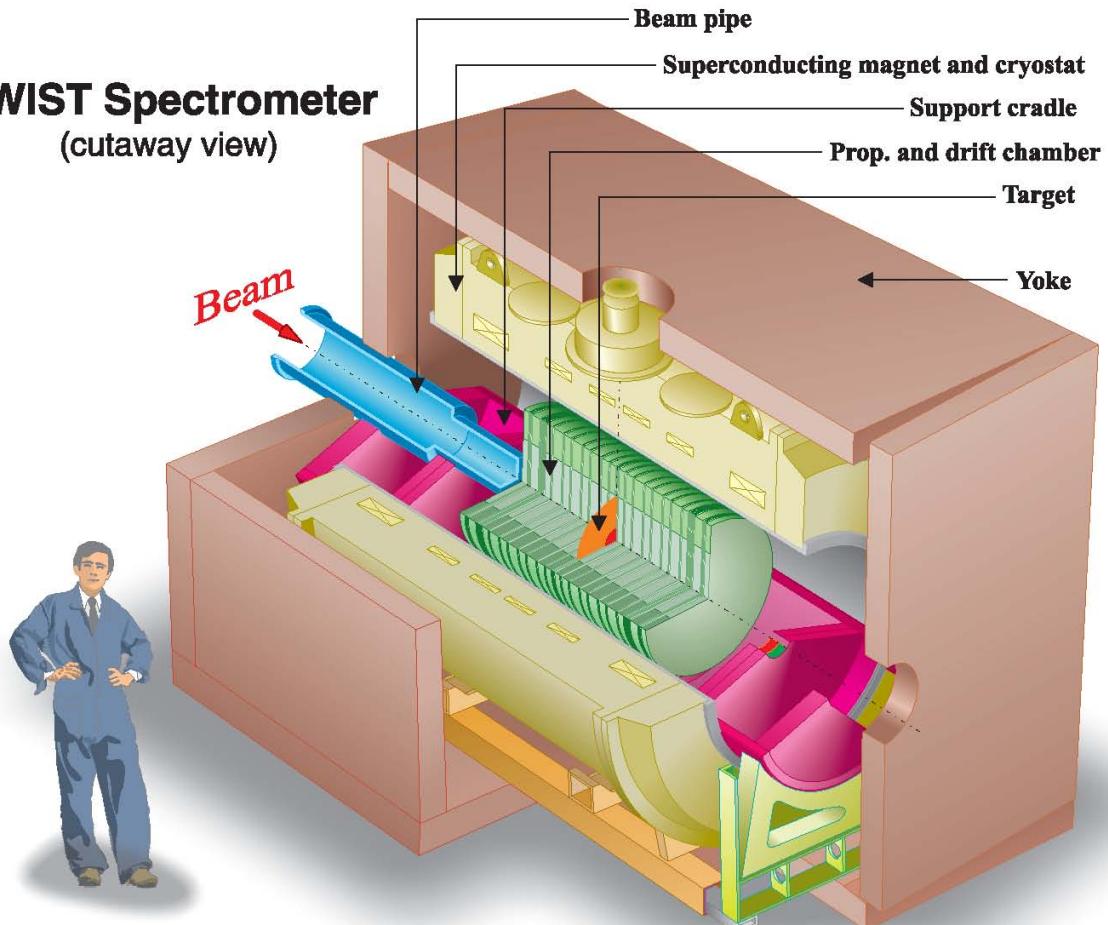
by one order of magnitude compared to prior experimental results.

→ measure yield vs. energy and angle, and understand depolarization, to a few parts in 10^4 .

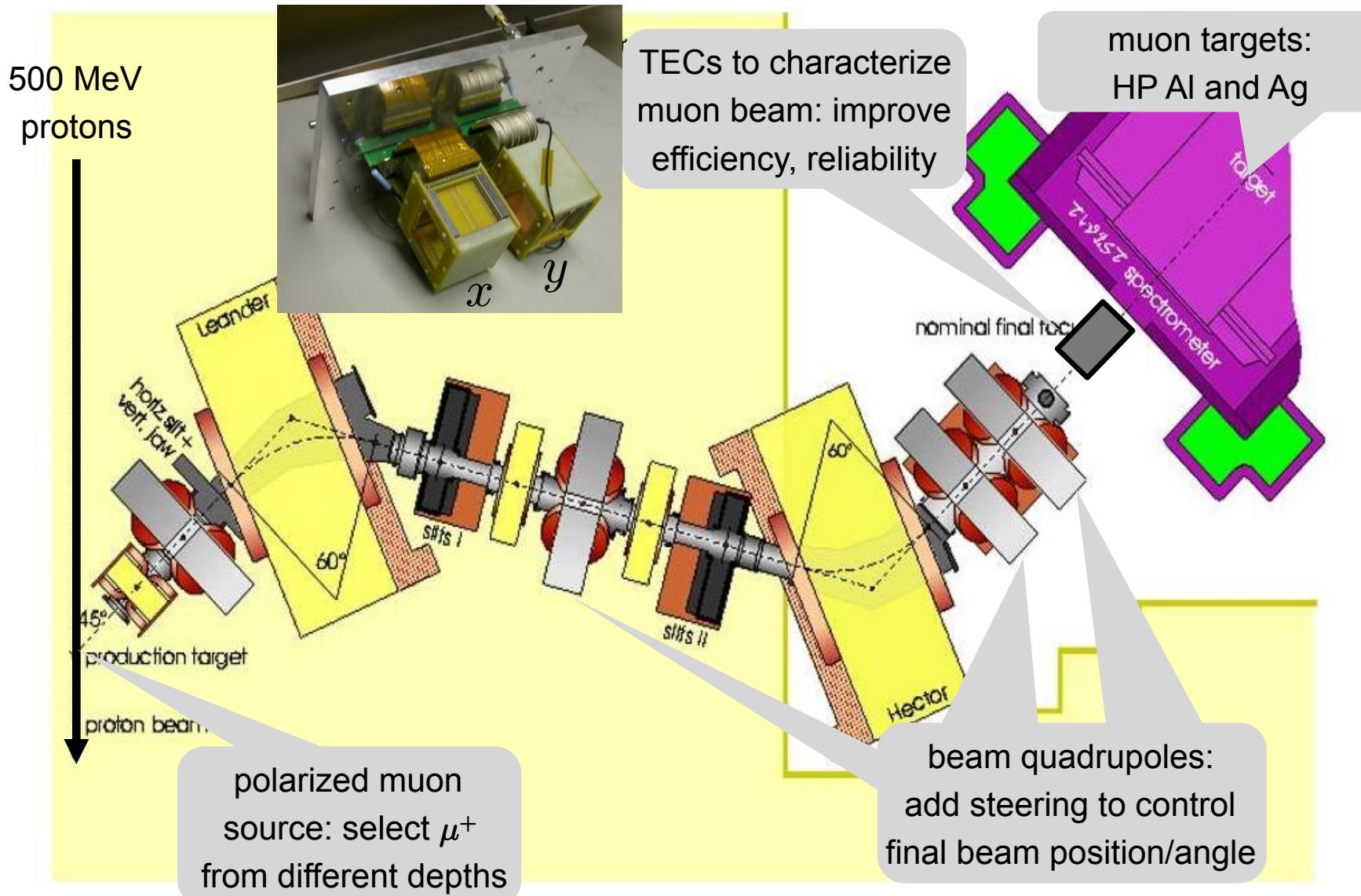
Spectrometer and muon target

- ▶ Highly polarized μ^+ beam
- ▶ μ^+ stop in a symmetric detector
- ▶ e^+ tracked through uniform, well-known field.
- ▶ Decay parameters found by comparison to detailed GEANT3 simulation.
- ▶ Data taking completed in 2007.

TWIST Spectrometer
(cutaway view)

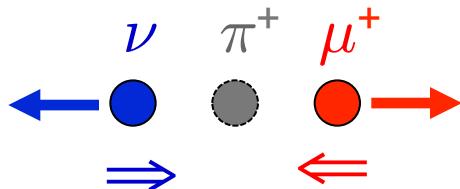


Muon production and transport

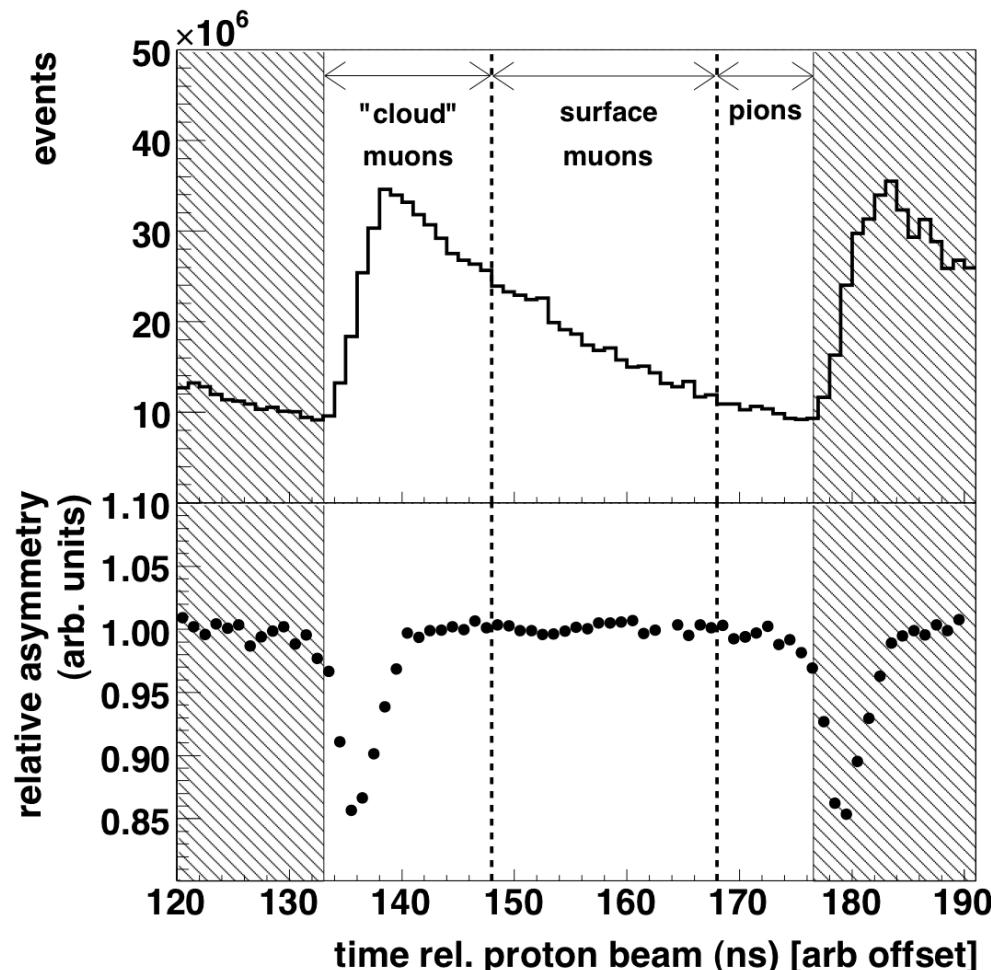


Muon beam polarization

- Pions decay to muons with $\mathcal{P}_\mu \pi \sim 100\%$ polarization antiparallel to muon momentum



- Control loss of polarization by
 - small solid angle
 - narrow momentum acceptance near 29.8 MeV/c
 - selected TOF with pion lifetime for pure surface muons (only at TRIUMF!)

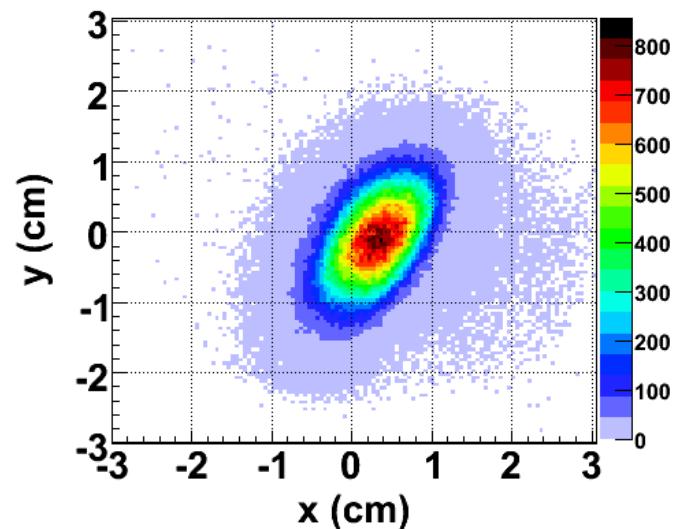
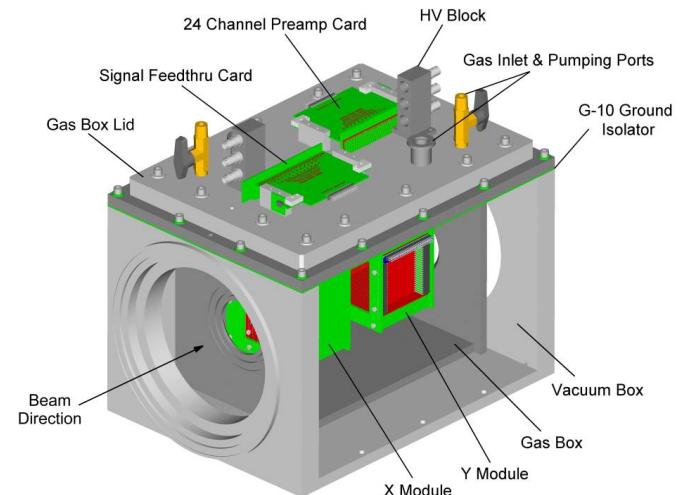


*TWIST Collaboration, J. Bueno et al.,
to be published*

TECs for beam characterization

- ▶ Need to know x , y , θ_x , θ_y , and correlations, for incident muon beam.
- ▶ Measure in two modules of low pressure (80 mbar) time expansion chambers (TEC).
- ▶ “Correct” for multiple scattering (~ 20 mrad rms).
- ▶ Simulate by sampling corrected distributions.
- ▶ Decay parameters measured with TEC removed; multiple scattering reduces polarization.

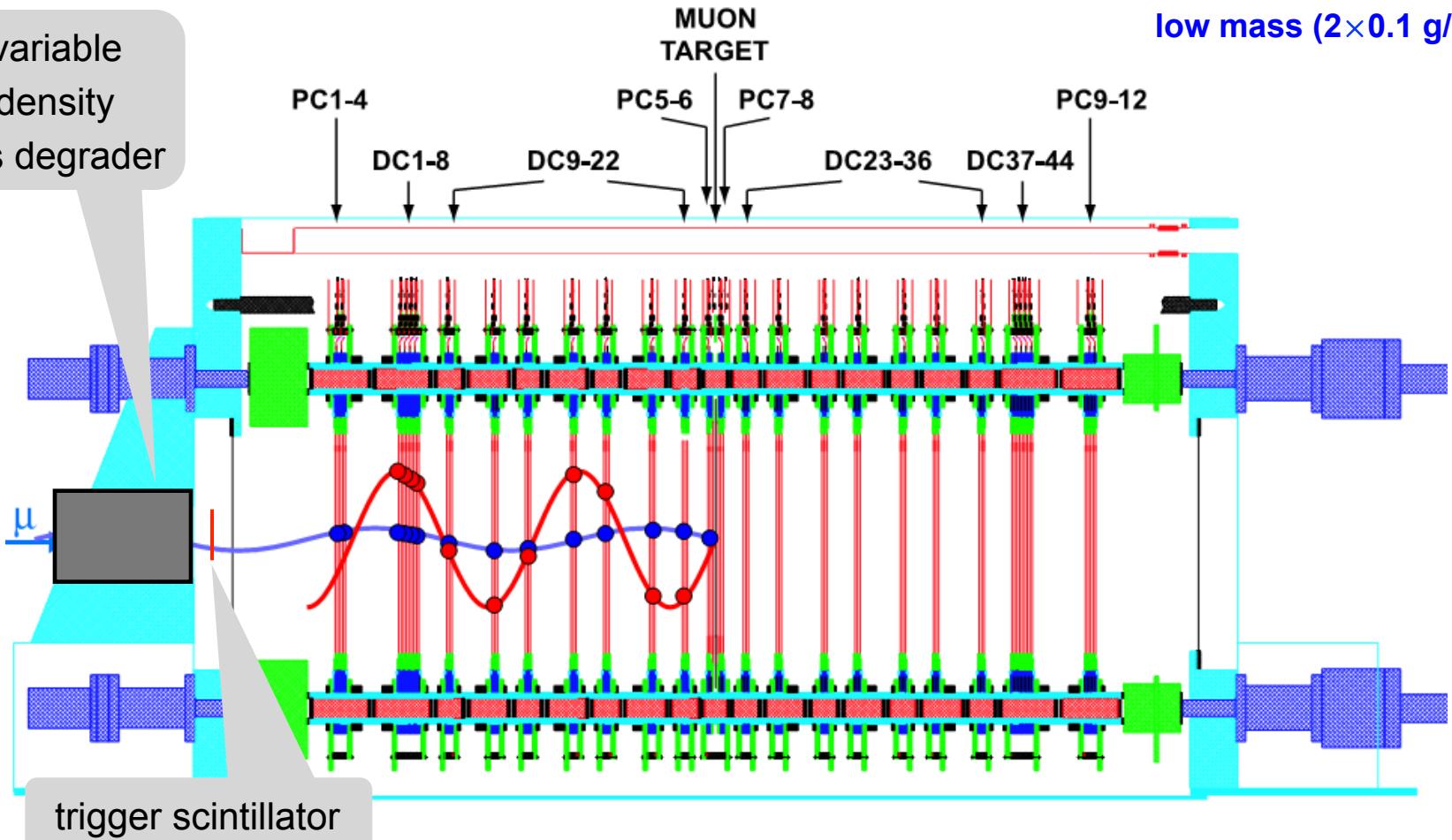
J. Hu et al., NIM A566, 563 (2006).



Detector array

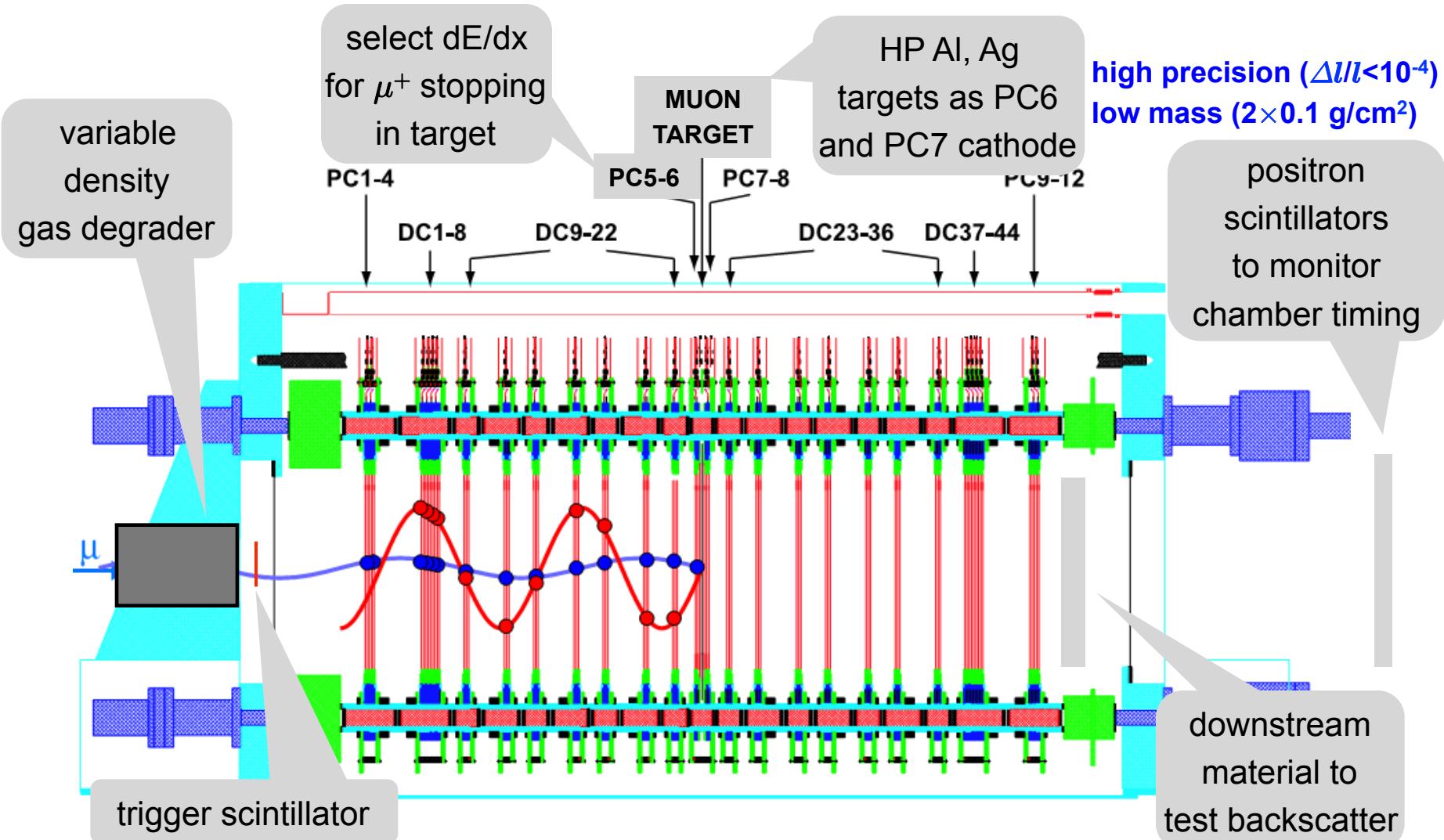
variable
density
gas degrader

high precision ($\Delta l/l < 10^{-4}$)
low mass ($2 \times 0.1 \text{ g/cm}^2$)



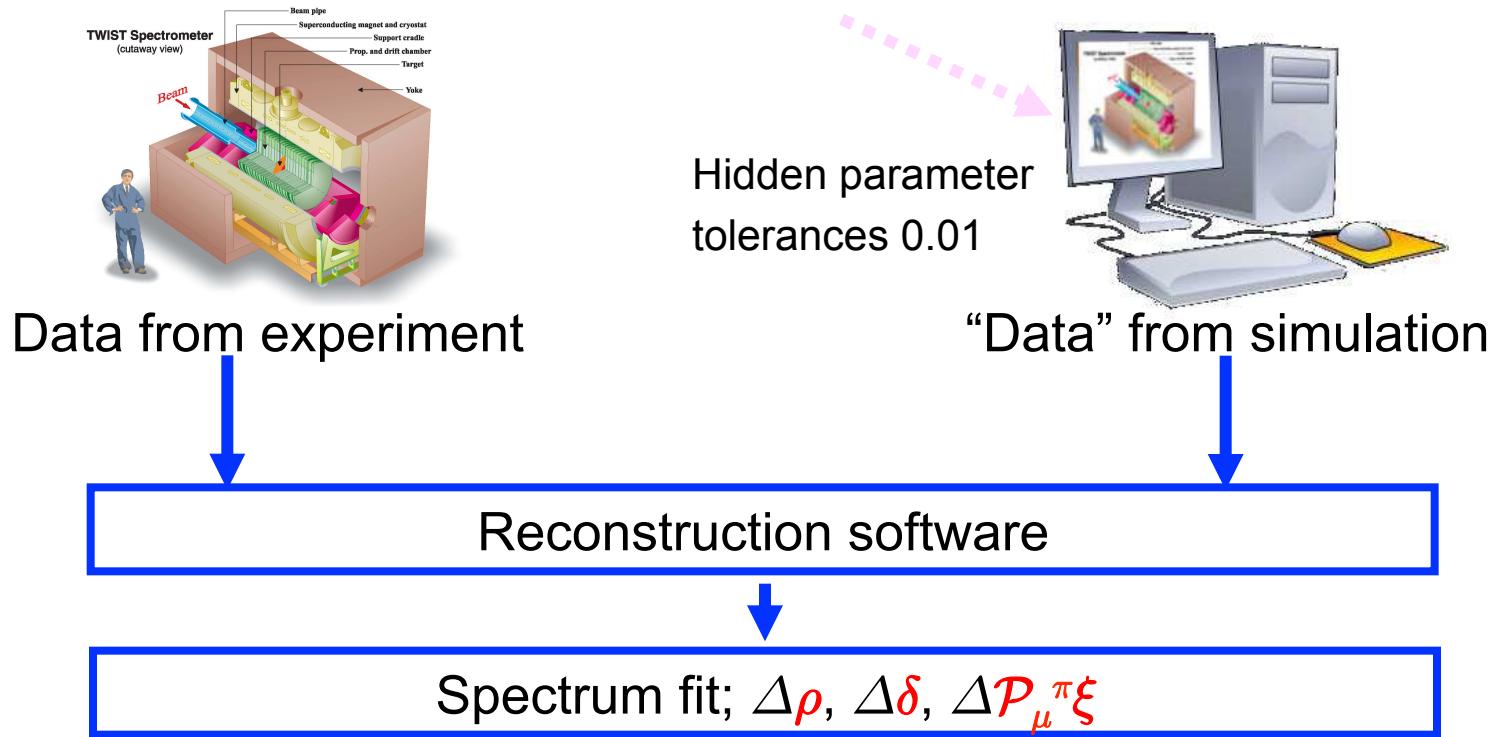
R. Henderson et al., Nucl. Instr. and Meth. A548, 306 (2005).

Detector array

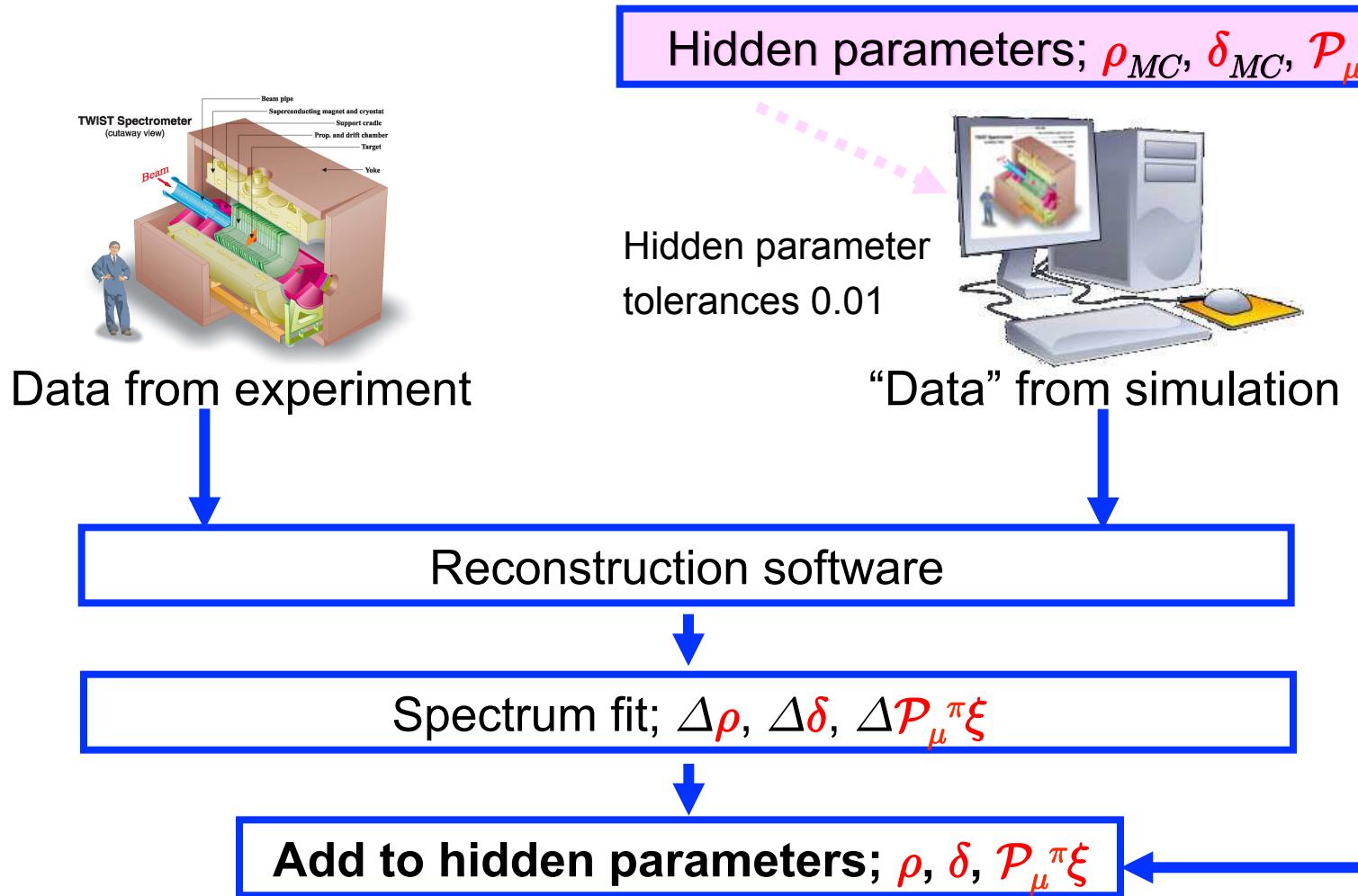


R. Henderson et al., Nucl. Instr. and Meth. A548, 306 (2005).

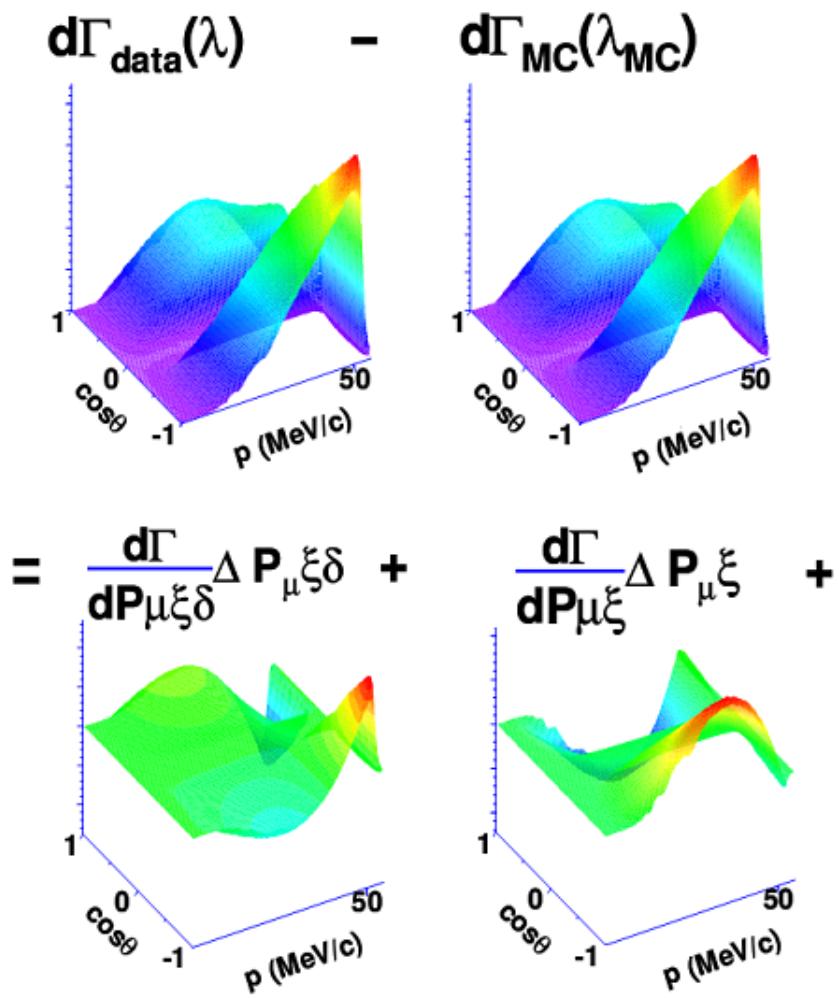
Analysis



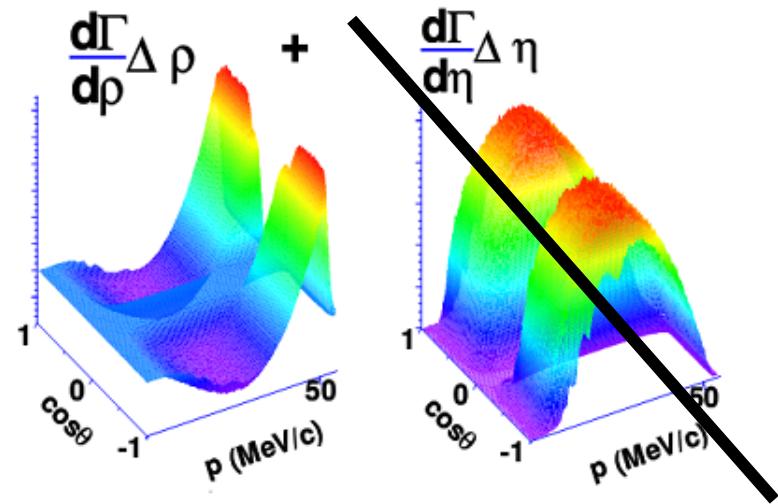
Analysis



Two-dimensional spectrum fit

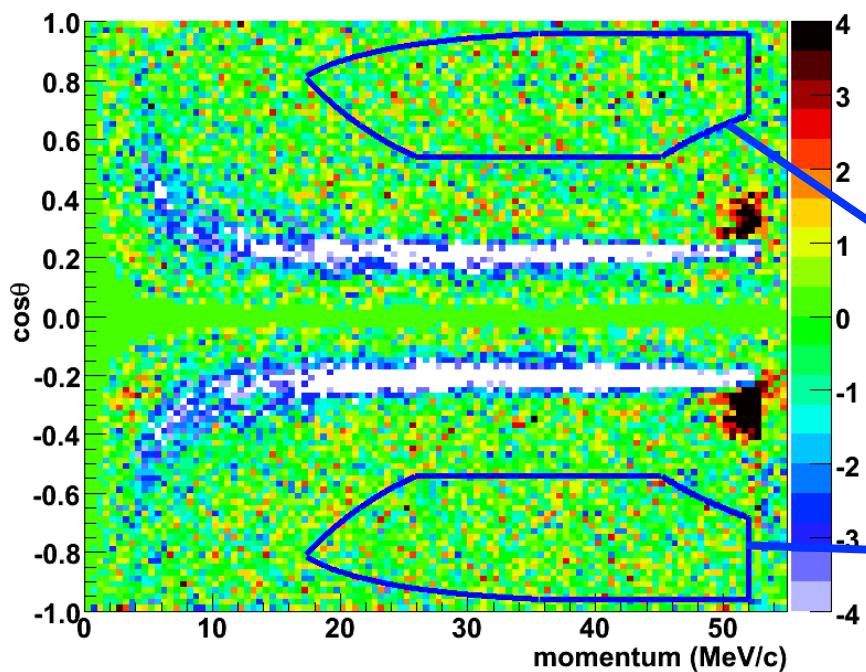


- ▶ fit data to normalized GEANT3 simulation
- ▶ use linearity in $P_\mu \xi$, $P_\mu \xi \delta$, ρ , η
- ▶ measure **differences** from hidden parameters λ_{MC} .

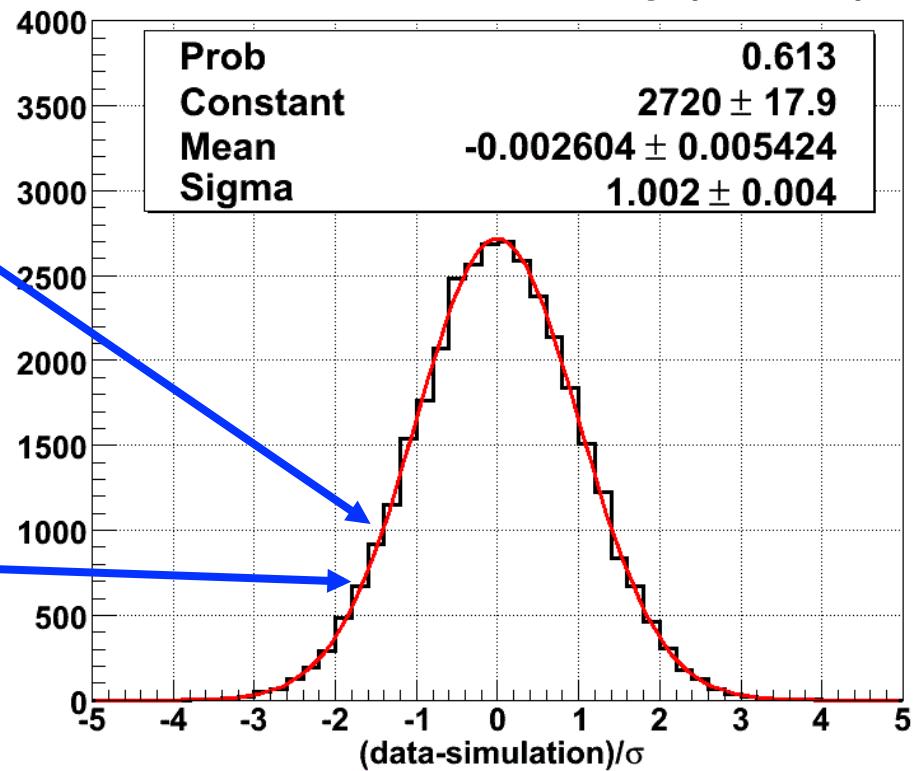


Spectrum fit quality

Normalised residuals for nominal set (s87)

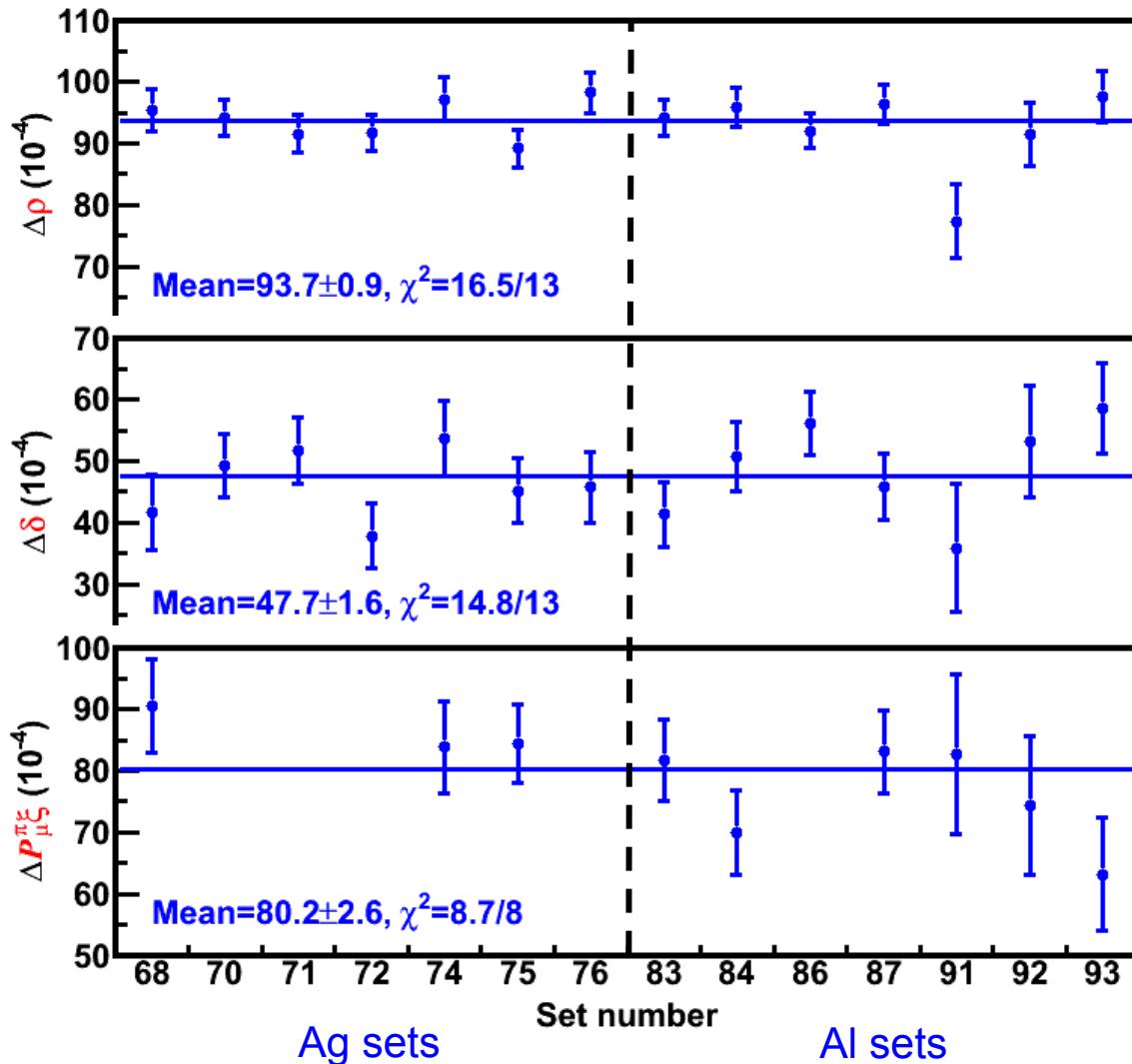


Residuals in fiducial only (all sets)



- ▶ Fiducial region: $p < 52.0 \text{ MeV}/c$, $0.54 < |\cos\theta| < 0.96$,
- ▶ $10.0 \text{ MeV}/c < p_T < 38.0 \text{ MeV}/c$, $|p_Z| > 14.0 \text{ MeV}/c$
- ▶ All data sets: 11×10^9 events, 0.55×10^9 in $(p, \cos\theta)$ fiducial
- ▶ Simulation sets: 2.7 times data statistics

Set-to-set statistical consistency



Key:

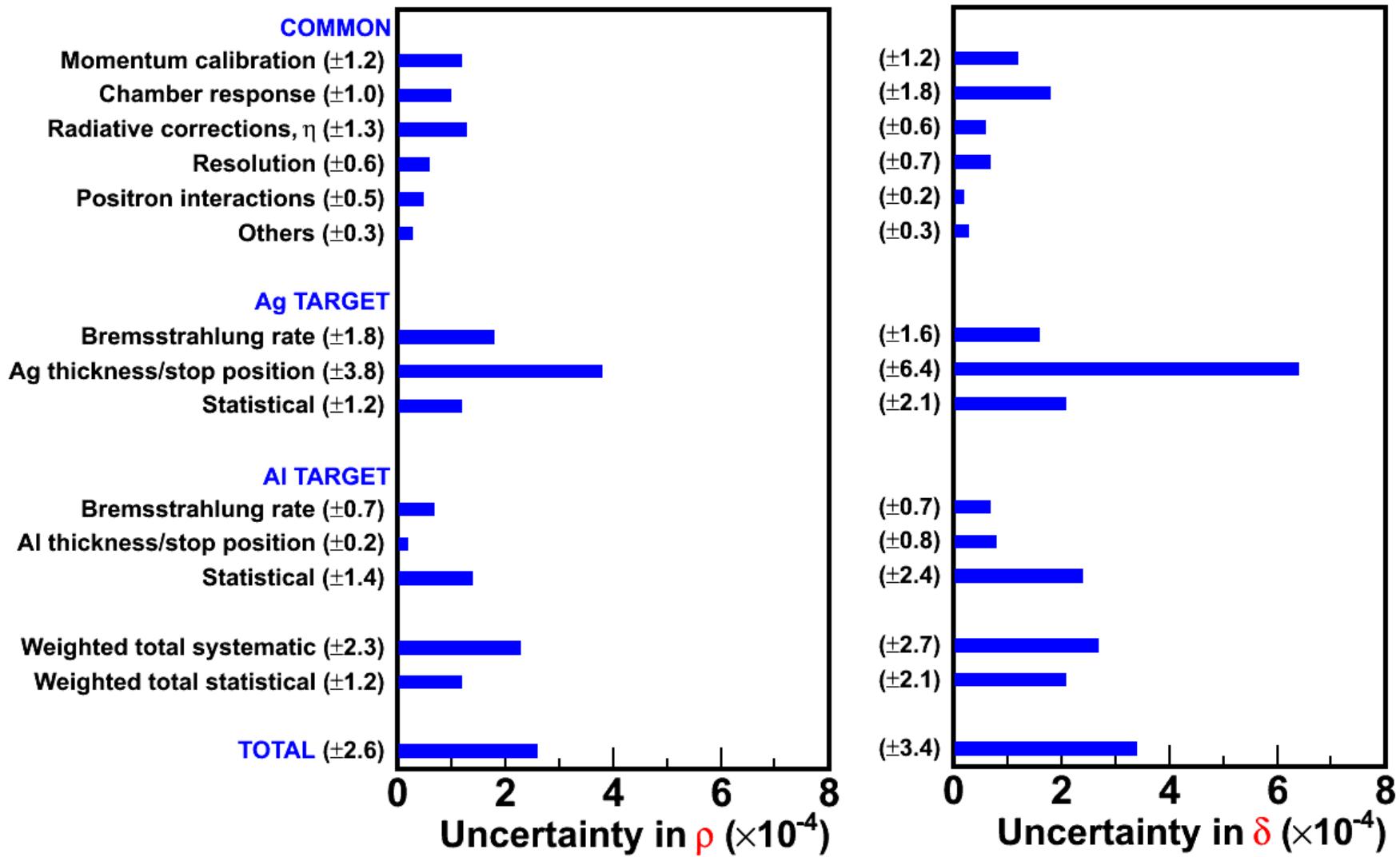
- ▶ **Ag target sets**
 - ▶ 68- μ stop slightly US
 - ▶ 70- $B = 1.96T$
 - ▶ 71- $B = 2.04T$
 - ▶ 72- TECs in
 - ▶ 74- production
 - ▶ 75- production
 - ▶ 76- μ beam mis-steered
- ▶ **Al target sets**
 - ▶ 83- DS extra material
 - ▶ 84- production
 - ▶ 86- μ beam mis-steered
 - ▶ 87- production
 - ▶ 91- low beam momentum
 - ▶ 92- low beam momentum
 - ▶ 93- low beam momentum

Differences (Δ) are with respect to blind parameters. Set-dependent corrections are applied; error bars and weights for the means are *statistical only*.

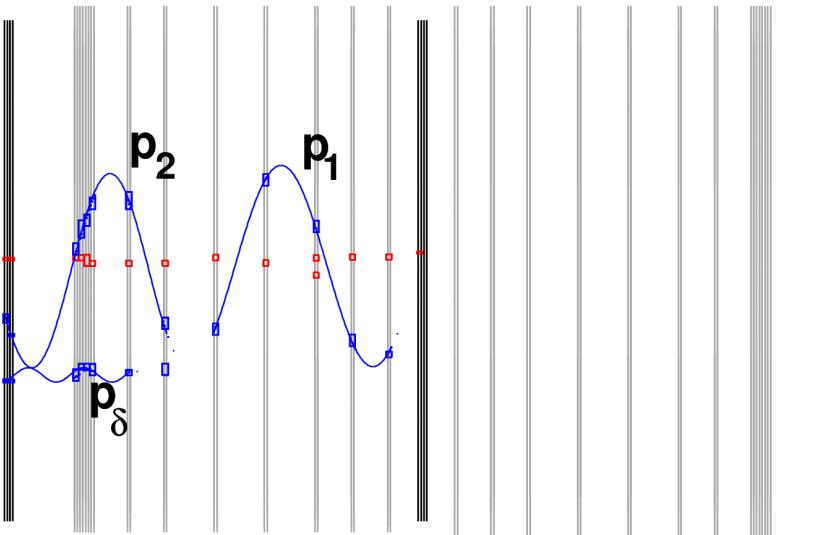
Blind vs. revised analysis

- ▶ The blind analysis results showed evidence of possible mistakes:
 - ▶ set-to-set statistical consistency satisfactory for ρ , δ , $\mathcal{P}_\mu^{\pi\xi}$
 - ▶ but $\mathcal{P}_\mu^{\pi\xi\delta}/\rho$ different for Al and Ag targets by 3.9σ .
 - ▶ $\mathcal{P}_\mu^{\pi\xi\delta}/\rho$ averaged over all sets was 2.9σ greater than 1.0.
 - ▶ unlikely in four-fermion formulation with massless neutrinos.
- ▶ Search for mistakes identified two corrections and two procedural changes:
 - ▶ radiative decay: small correction for Ag only
 - ▶ mean stopping position differences (data vs. simulation): corrected set-by-set, based on better analysis of stop position
 - ▶ separate systematic uncertainties for Ag and Al targets for bremsstrahlung, target thickness, and mean stopping position
 - ▶ ρ and δ correlations from all sets applied to $\mathcal{P}_\mu^{\pi\xi}$
- ▶ After the revisions, the Ag-Al $\mathcal{P}_\mu^{\pi\xi\delta}/\rho$ difference becomes $<1\sigma$.

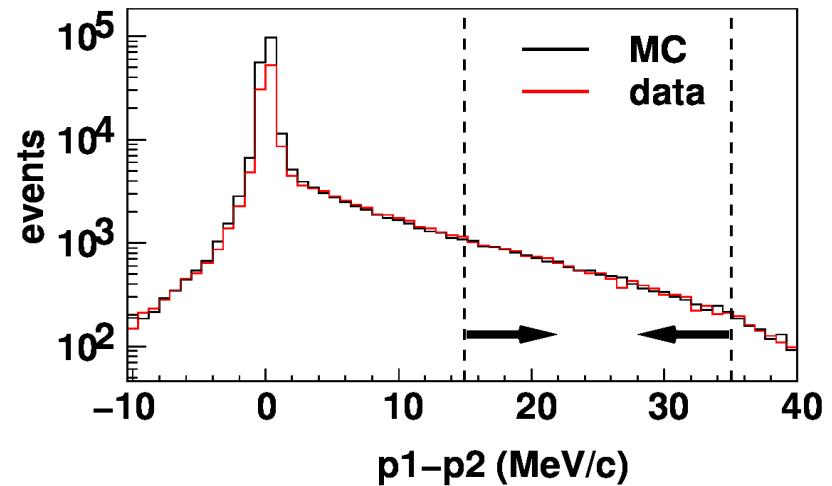
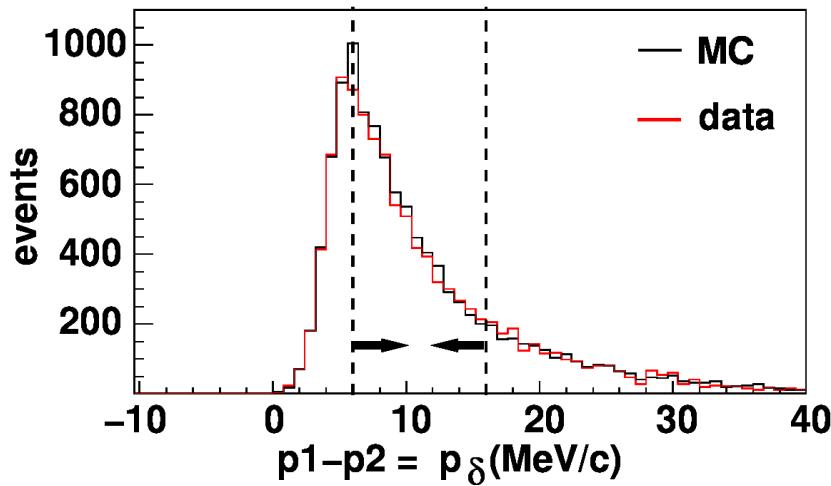
Uncertainties in ρ and δ



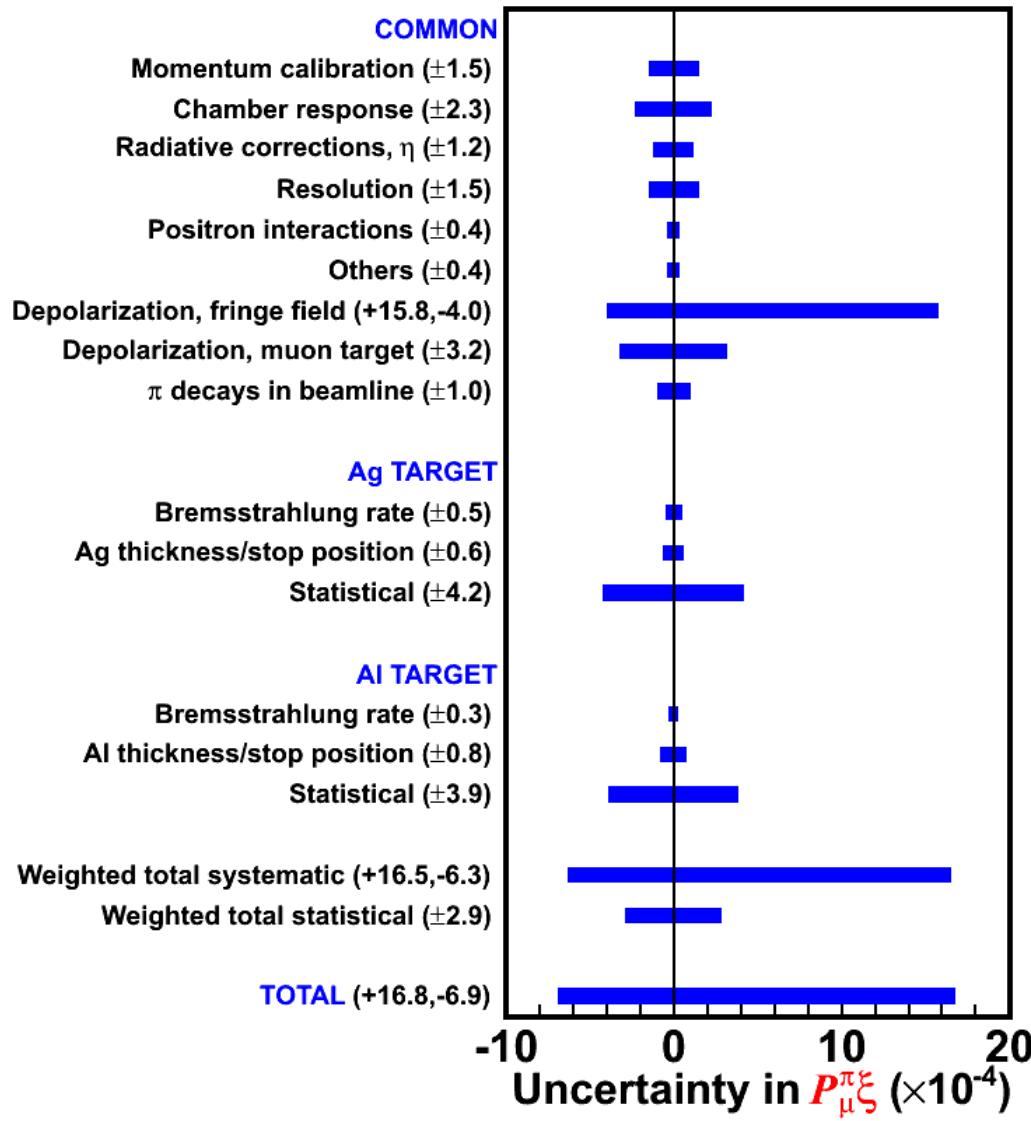
Positron interactions systematic



- ▶ “Broken tracks” analysis:
 - ▶ $2 e^+, 1 e^- \rightarrow \delta$ -electron
 - ▶ $2 e^+ \rightarrow$ Bremsstrahlung
- ▶ Agreement of data and sim:
 - ▶ δ -electrons < 1%
 - ▶ Bremsstrahlung differs by 2.4%

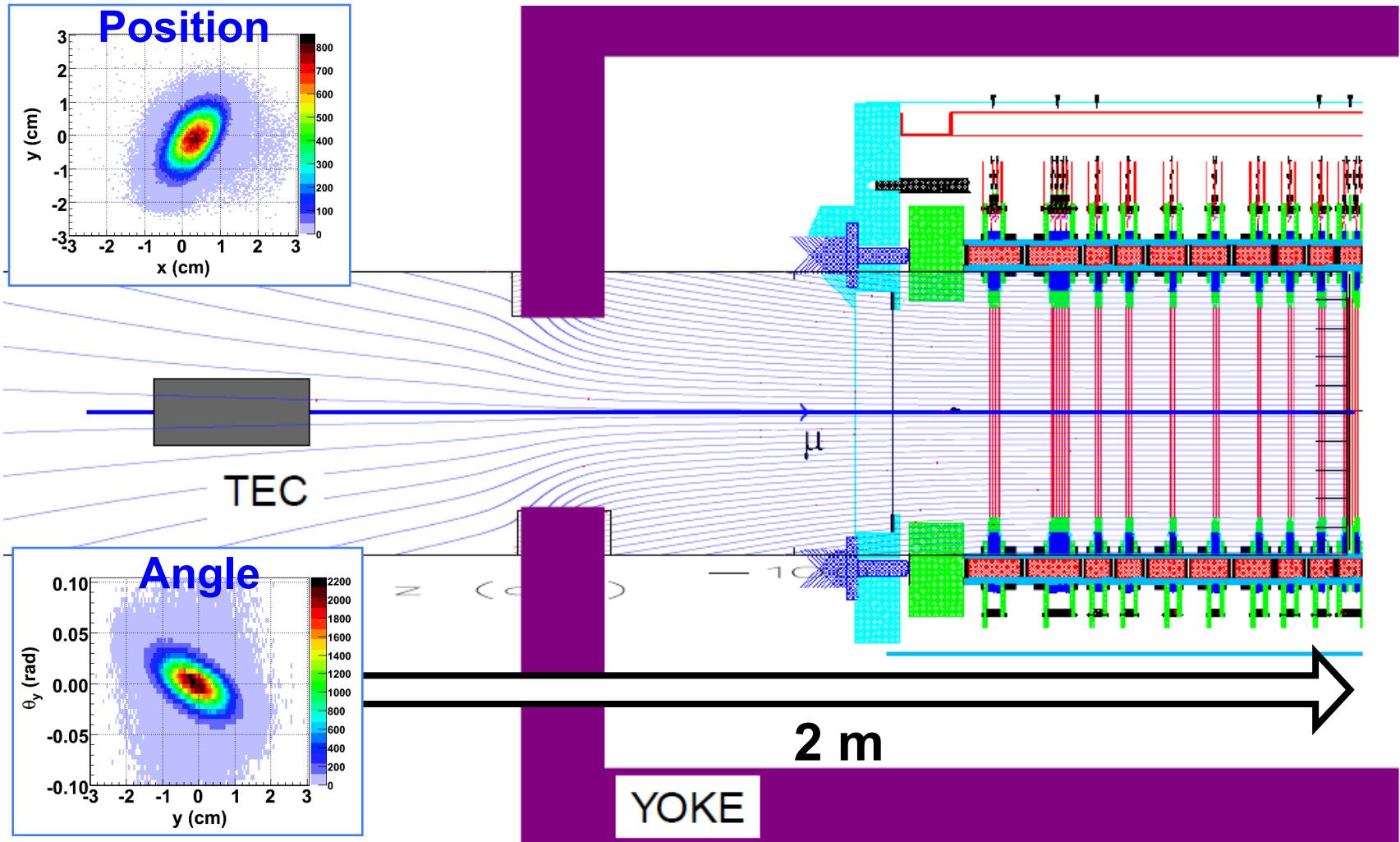


Uncertainties in $\mathcal{P}_\mu \pi \xi$

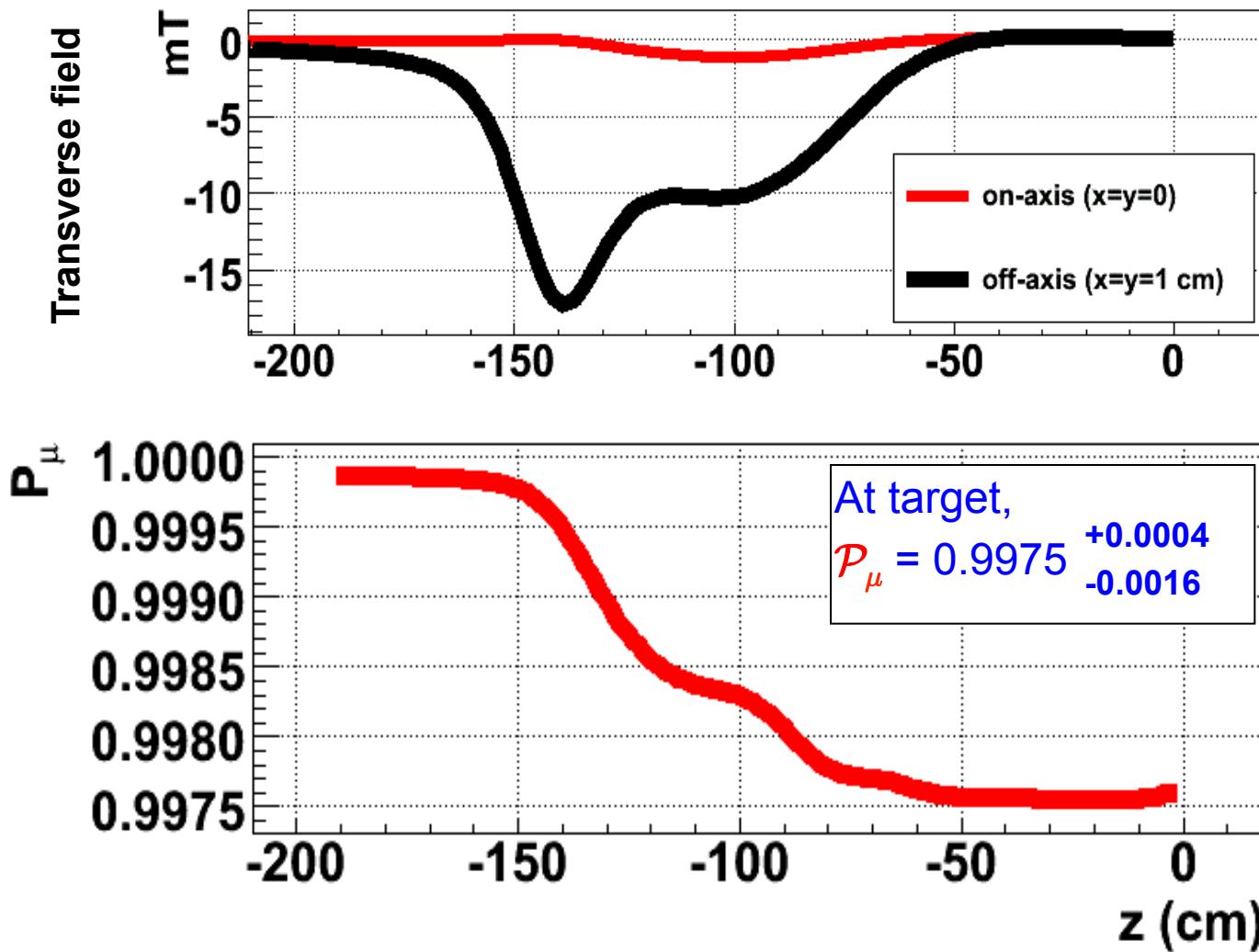


- ▶ Uncertainties for all three parameters are from the revised analysis
- ▶ Differences to blind results are small:
 - ▶ $\sigma(\rho)$ changed by -0.3×10^{-4}
 - ▶ $\sigma(\delta)$ changed by $+0.1 \times 10^{-4}$
 - ▶ $\sigma(\mathcal{P}_\mu \pi \xi_{\text{avg}})$ changed by -0.2×10^{-4}
- ▶ Difference of $\mathcal{P}_\mu \pi \xi \delta / \rho$ for Ag and Al is reduced to $< 1\sigma$ in the revised analysis.

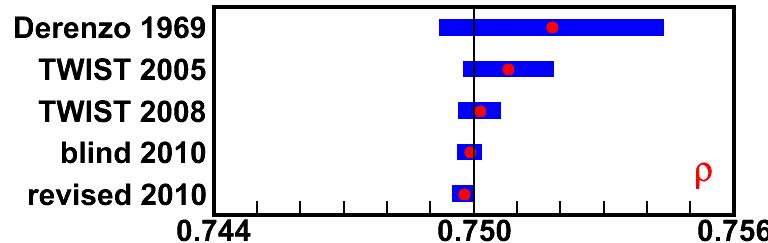
Fringe field depolarization



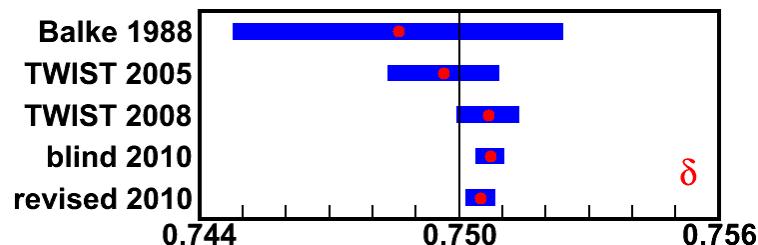
Asymmetric depolarization systematic



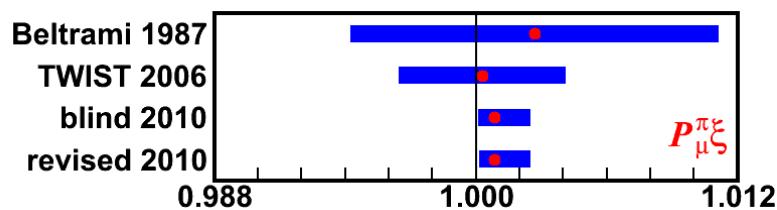
Decay parameter results



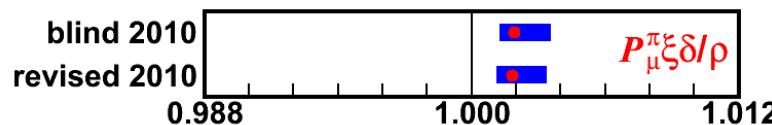
$\rho = 0.74977 \pm 0.00012 \text{ (stat)} \pm 0.00023 \text{ (syst)}$
 $(<1\sigma \text{ from SM, } -1.4 \times 10^{-4} \text{ from blind})$



$\delta = 0.75049 \pm 0.00021 \text{ (stat)} \pm 0.00027 \text{ (syst)}$
 $(+1.4\sigma \text{ from SM, } -2.3 \times 10^{-4} \text{ from blind})$



$P_\mu^{\pi\xi} = 1.00084 \pm 0.00029 \text{ (stat)} {}^{+0.00165}_{-0.00063} \text{ (syst)}$
 $(+1.2\sigma \text{ from SM, same as blind})$



$P_\mu^{\pi\xi\delta/\rho} > 0.99909 \text{ (90\%CL)}$
from global analysis

TWIST Collaboration, R. Bayes et al., Phys. Rev. Lett. 106, 041804 (2011).

Left-right symmetric analysis

- Heavy W_R that mixes with W_L to restore parity at high energy

$$W_L = W_1 \cos \zeta + W_2 \sin \zeta, \quad W_R = e^{i\omega} (-W_1 \sin \zeta + W_2 \cos \zeta)$$

- P. Herczeg, PRD 34, 3499 (1986) uses general parameters:

$$t = \frac{g_R^2 m_1^2}{g_L^2 m_2^2}, \quad t_\theta = t \frac{|V_{ud}^R|}{|V_{ud}^L|} \simeq t \frac{\cos \theta_R}{\cos \theta_{Cab}}, \quad \zeta_g^2 = \frac{g_R^2}{g_L^2} \zeta^2$$

- g_L , g_R and V_{ud}^L , V_{ud}^R permit differences in left and right sectors, with possible CP violating phases ω and α , and for muon decay:

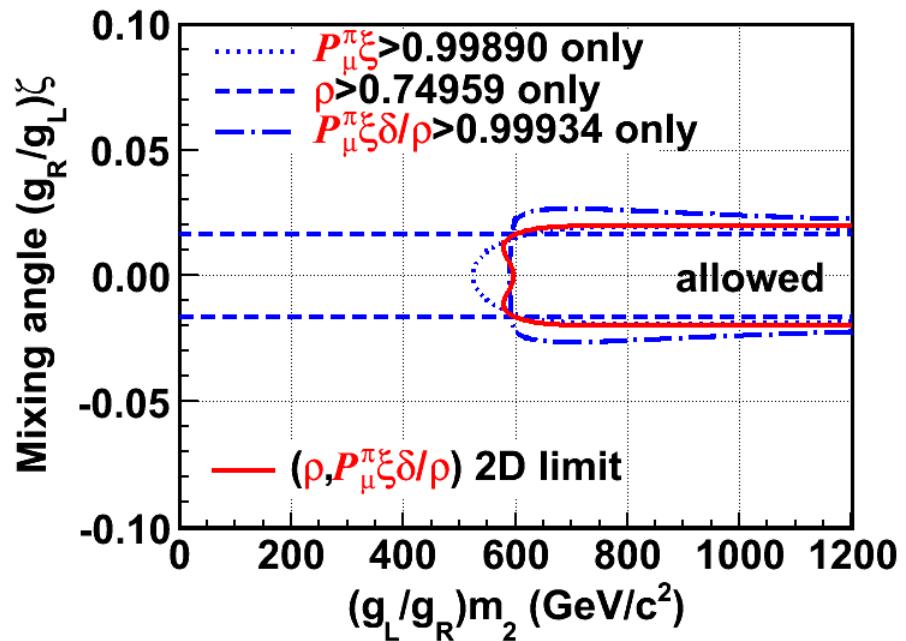
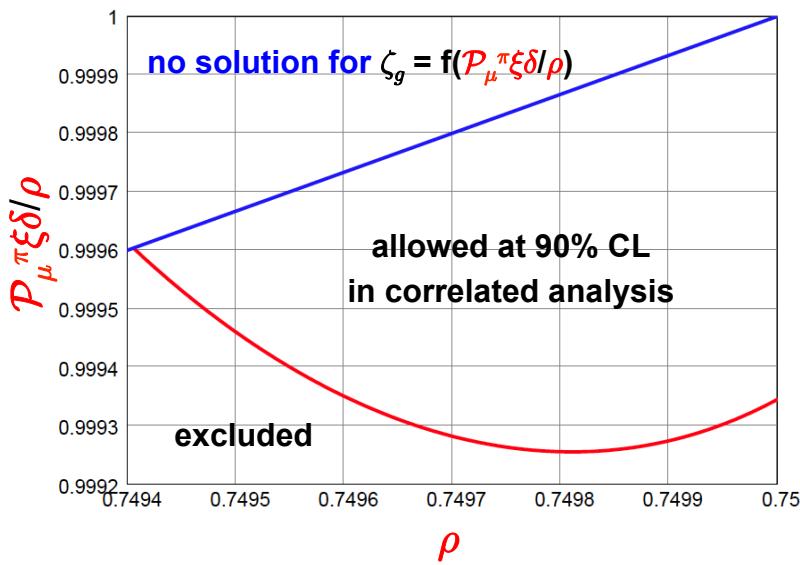
$$\rho \simeq \frac{3}{4}(1 - 2\zeta_g^2), \quad \delta = \frac{3}{4}, \quad \xi \simeq 1 - 2(t^2 + \zeta_g^2),$$

$$\mathcal{P}_\mu^\pi \simeq 1 - 2t_\theta^2 - 2\zeta_g^2 - 4t_\theta \zeta_g \cos(\alpha + \omega)$$

- allowing restrictions to be put on LRS mass m_2 and mixing ζ , e.g.,

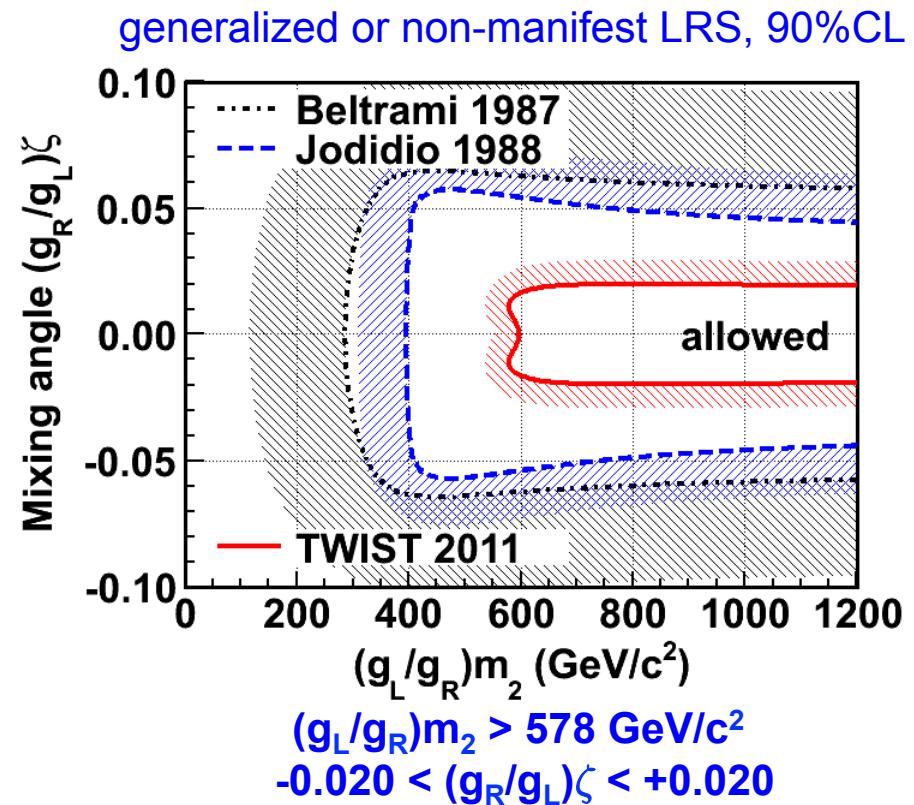
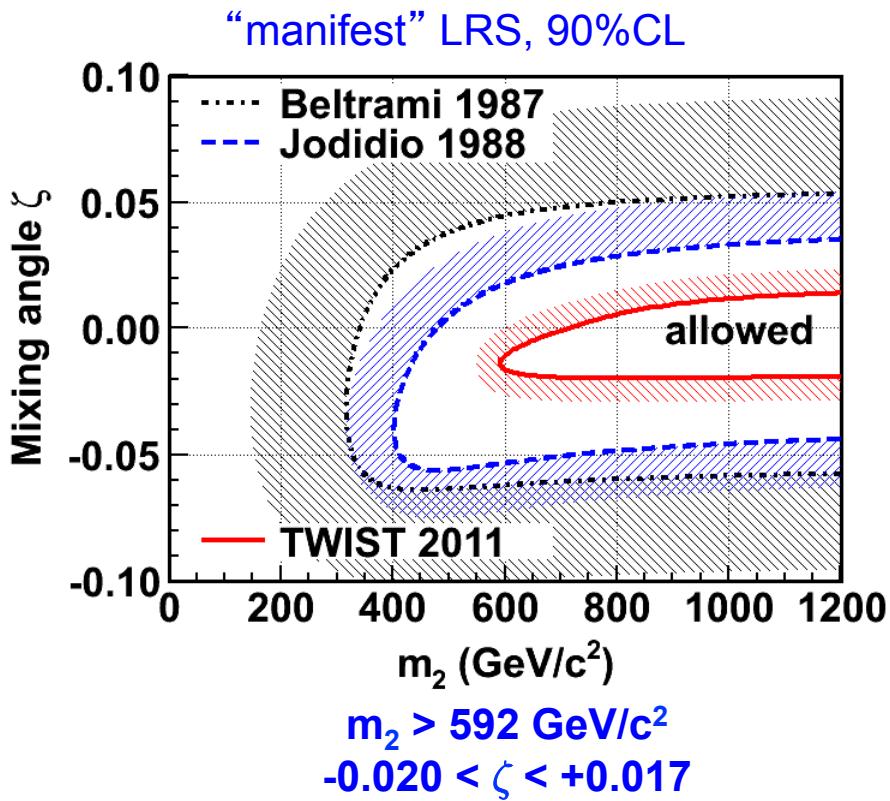
$$1 - \frac{\mathcal{P}_\mu^\pi \xi \delta}{\rho} \simeq 2t^2 \left(1 + \frac{\cos^2 \theta^R}{\cos^2 \theta_{Cab}}\right) + 2\zeta_g^2 + 4\zeta_g t \frac{\cos \theta^R}{\cos \theta_{Cab}} \cos(\alpha + \omega)$$

TWIST 2D exclusion plot and LRS limits



- ▶ Previous muon decay LRS parameter limits used individual limits for ρ , $P_\mu^{\pi\xi}$, or $P_\mu^{\pi\xi\delta}/\rho$.
- ▶ *TWIST* has simultaneous measurements of three parameters; correlations contribute to the confidence interval.

Left-Right Symmetric limit comparison



- ▶ Other W' direct search mass limits
 - ▶ ATLAS: >1.49 TeV/c², 95%CL (LLWI11)
 - ▶ CMS: >1.58 TeV/c², 95%CL (LLWI11)
 - ▶ CMS: >1.36 TeV/c², 95%CL (2011)
 - ▶ CDF: >1.12 TeV/c², 95%CL (2011)
 - ▶ D0: >1.0 TeV/c², 95%CL (2008)

- ▶ Other limits on mixing angle ζ
 - ▶ Hardy and Towner: <0.0005 (MLRS), <0.04 (generalized)
 - ▶ K decay: <0.004 (MLRS)

Global analysis result

- ▶ Include new results with other muon decay observables to restrict coupling constants
 - ▶ influences mostly right-handed muon terms

$$\begin{aligned} Q_R^\mu &= \frac{1}{4}|g_{LR}^S|^2 + \frac{1}{4}|g_{RR}^S|^2 + |g_{LR}^V|^2 + |g_{RR}^V|^2 + 3|g_{LR}^T|^2 \\ &= \frac{1}{2}\left[1 + \frac{1}{3}\xi - \frac{16}{9}\xi\delta\right] \\ &< 8.2 \times 10^{-4} \quad (90\% \text{C.L.}) \end{aligned}$$

- ▶ $\sim 6 \times$ reduction
- ▶ SM prediction is zero

Limits for heavy sterile neutrinos

- ▶ Muon decay spectrum shape places limits on heavy neutrino mass and mixing in a mass region inaccessible with π or K decays.

R.R. Schrock, Phys. Rev. D 24, 1275 (1981).

P. Kalyniak and J.N. Ng,
Phys. Rev. D 25, 1305 (1982).

M.S. Dixit et al., Phys. Rev. D 27, 2216 (1983).

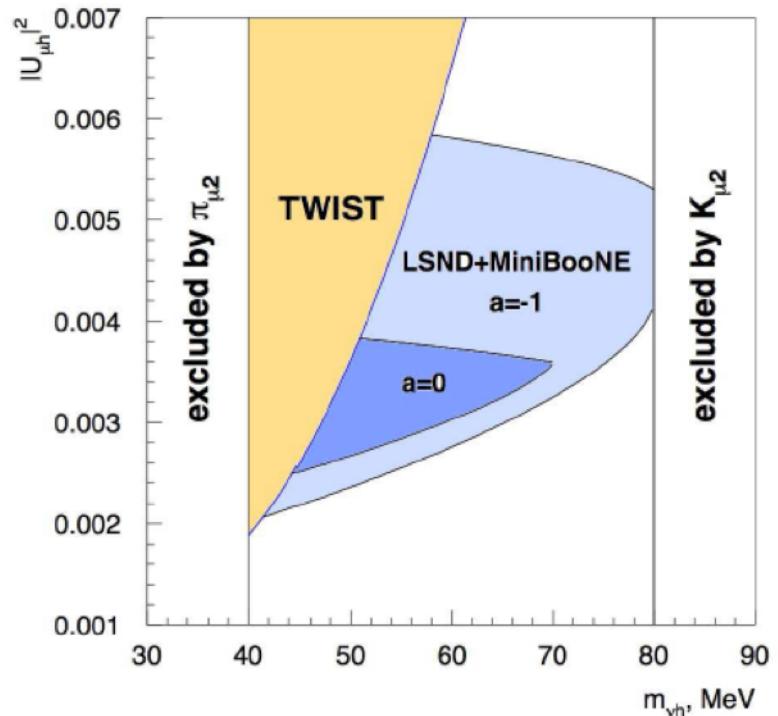


FIG. 24: The 2σ allowed region (dark areas) in the $(m_{\nu_h}; |U_{\mu h}|^2)$ parameter space obtained for different values of the asymmetry parameter a from the combined analysis of LSND and MiniBooNE ν_μ and $\bar{\nu}_\mu$ data. The areas excluded by the $\pi_{\mu 2}$ and $K_{\mu 2}$ decay experiments [45], and the exclusion region obtained in the present work from the results of precision measurements of the muon decay parameters by the TWIST experiment [50] are also shown; see Sec. VI.

Heavy sterile neutrino model

S.N. Gninenco, arXiv:1009.5536v3, Jan 2011

Summary

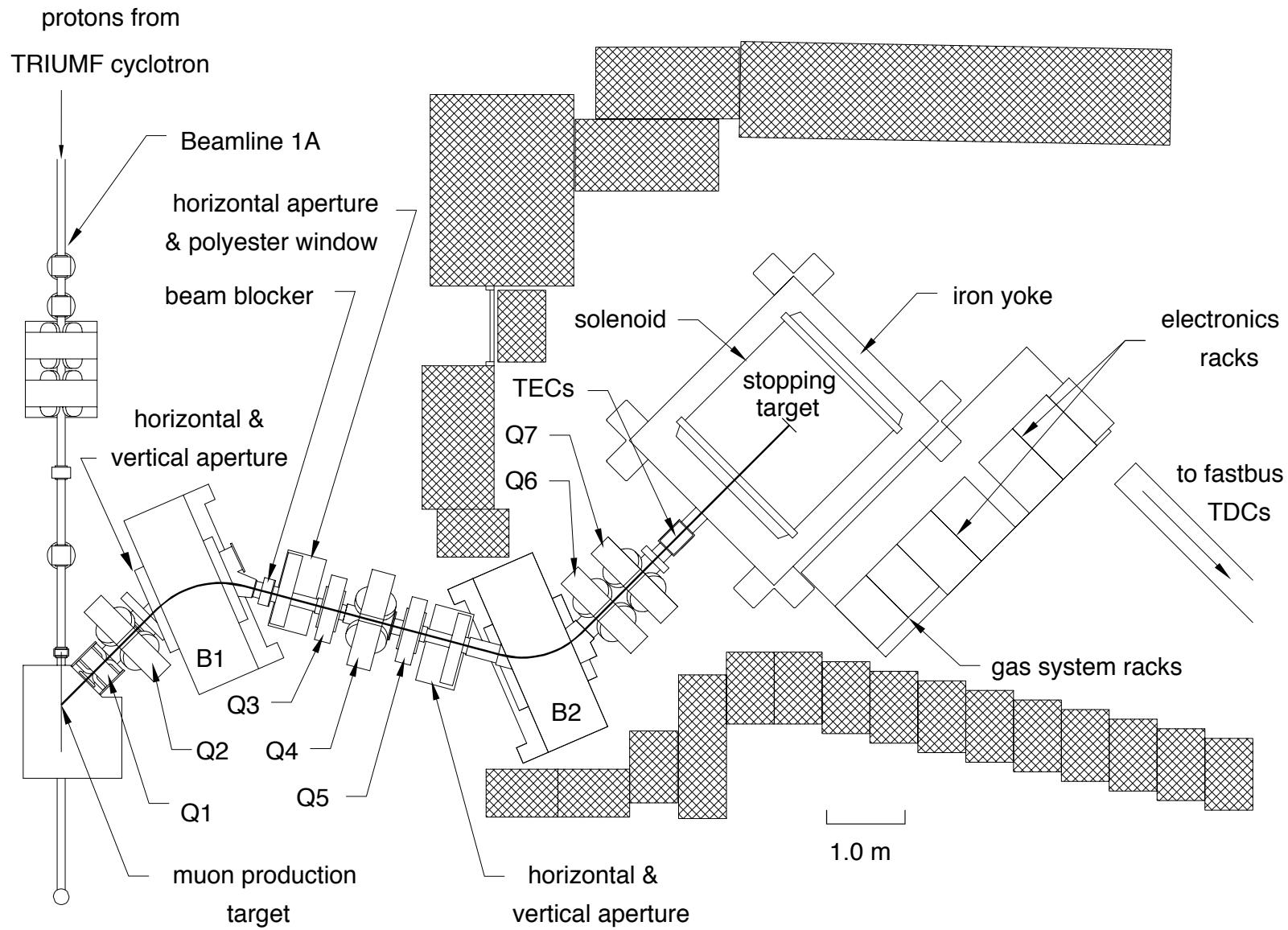
- ▶ Systematic uncertainties in muon decay parameter measurements were substantially reduced in *TWIST*.
- ▶ Total uncertainties were reduced by factors of **10**, **11**, and **7** for ρ , δ , and $\mathcal{P}_\mu \pi \xi$ respectively, roughly achieving the goals of the experiment.
- ▶ Differences with Standard Model predictions are respectively **-0.9 σ** , **+1.4 σ** , and **+1.2 σ** , after post-blind revisions.
- ▶ $\mathcal{P}_\mu \pi \xi \delta / \rho$ deviates by **+2.3 σ** from the expected upper limit of 1.0.

TWIST recent graduates



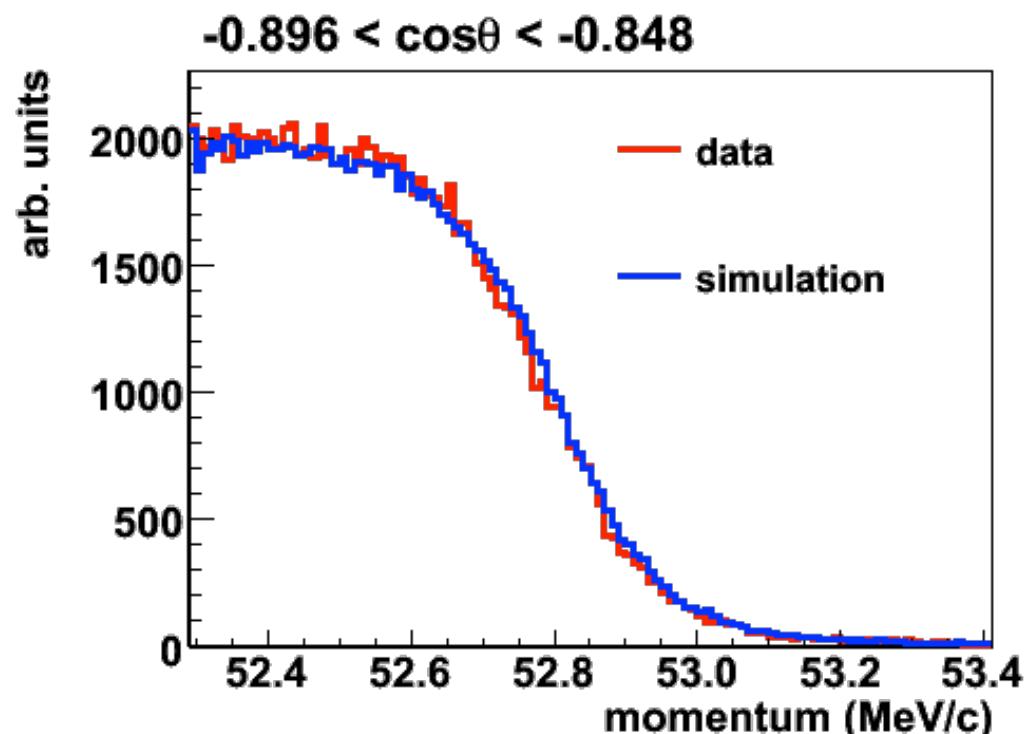
James Bueno, Ryan Bayes, and Anthony Hillairet

Proton and muon beams



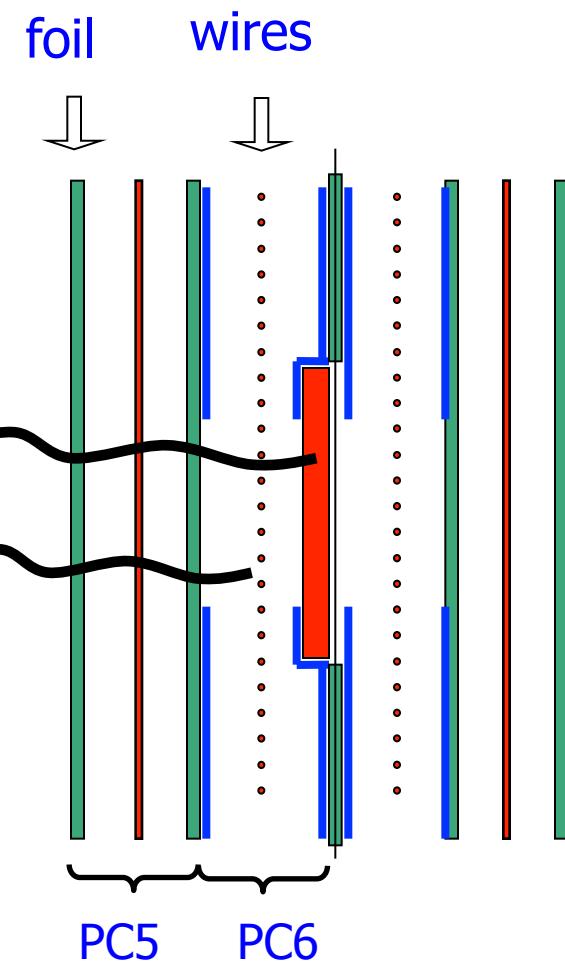
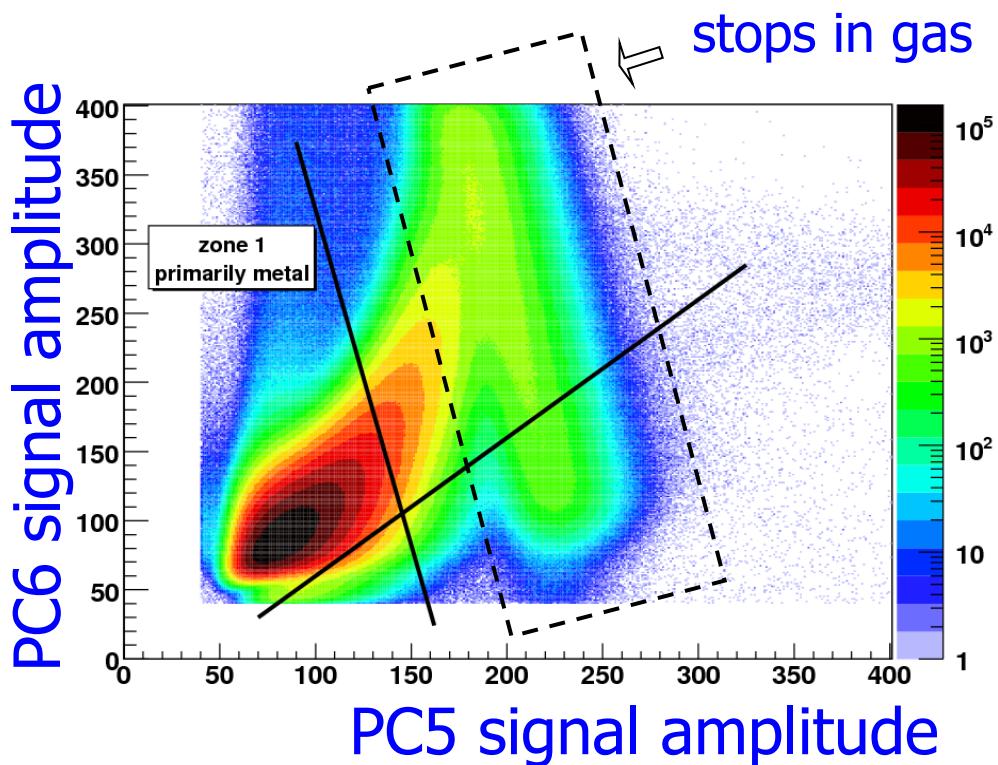
Momentum calibration

- ▶ Use kinematic edge at 52.8 MeV/c: energy loss and planar geometry lead to $\cos\theta$ dependence.
- ▶ Difference of ~ 10 keV/c prior to calibration.
- ▶ Calibration at edge provides no guidance on how to propagate the difference to lower momenta in the spectrum.



Selecting muons in metal target

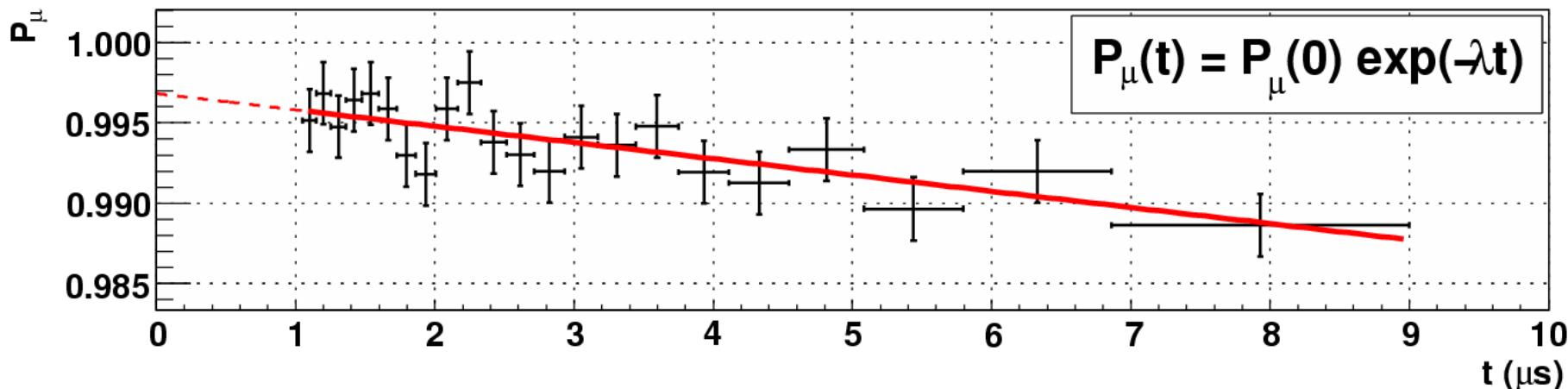
- Place cut on 2-d distribution so that <0.5% of “stops in gas” contaminate “stops in target” region (zone 1).



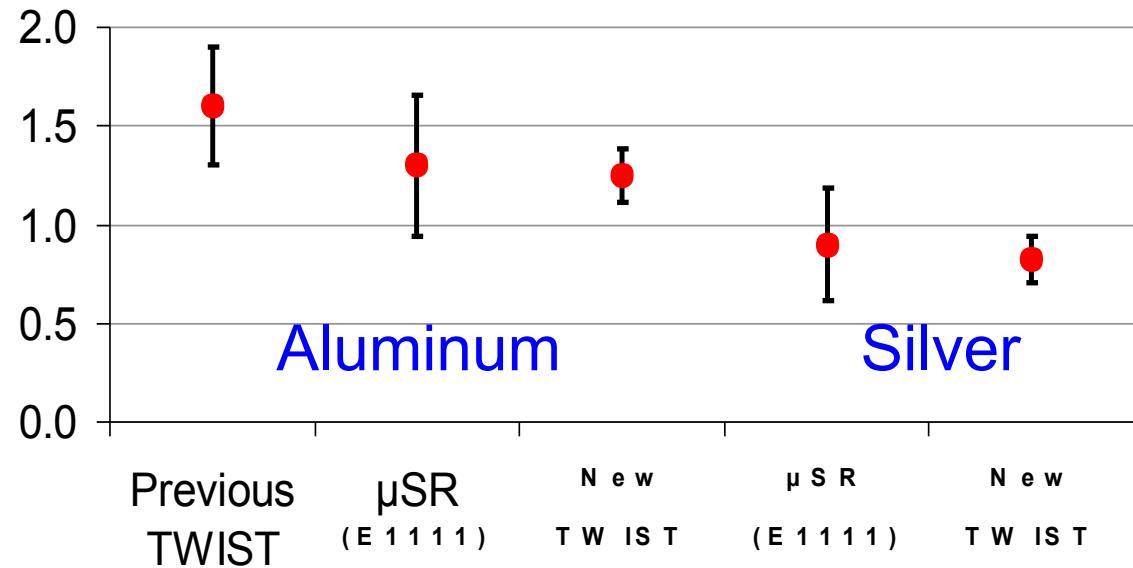
Corrections to fit results

- ▶ Depolarization from scattering in production target
 - ▶ $+0.9 \times 10^{-4}$ for full momentum sets, $+5.6 \times 10^{-4}$ for reduced momentum sets, for $\mathcal{P}_\mu \xi$ only.
- ▶ Simulations generated with incorrect polarization relaxation rates
 - ▶ $+2.9 \times 10^{-4}$ for Ag sets, $+2.4 \times 10^{-4}$ for Al sets
- ▶ Statistical biases
 - ▶ χ^2 fitting of Poisson statistics with $1/N$ weight is biased
 - ▶ in fitting data to simulation, weight includes $1/N$ from both
 - ▶ for unequal statistics, this is biased by $\sim 0.5 \times 10^{-4}$
 - ▶ energy calibration fit bias of typically $(-1.1, -0.4, +1.9) \times 10^{-4}$ for $\rho, \delta, \mathcal{P}_\mu \xi$, applied set-by-set

Depolarization in muon target material



- ▶ Estimate of relaxation is included in simulation; small correction is made to polarization parameter.
- ▶ μ S \mathcal{R} experiment establishes no fast relaxation.
- ▶ Statistical uncertainty in λ is included in decay parameter statistical uncertainty.



Spectrum fit quality

Normalised residuals for nominal set (s87)

