Positron energy calibration in the *TWIST* experiment.

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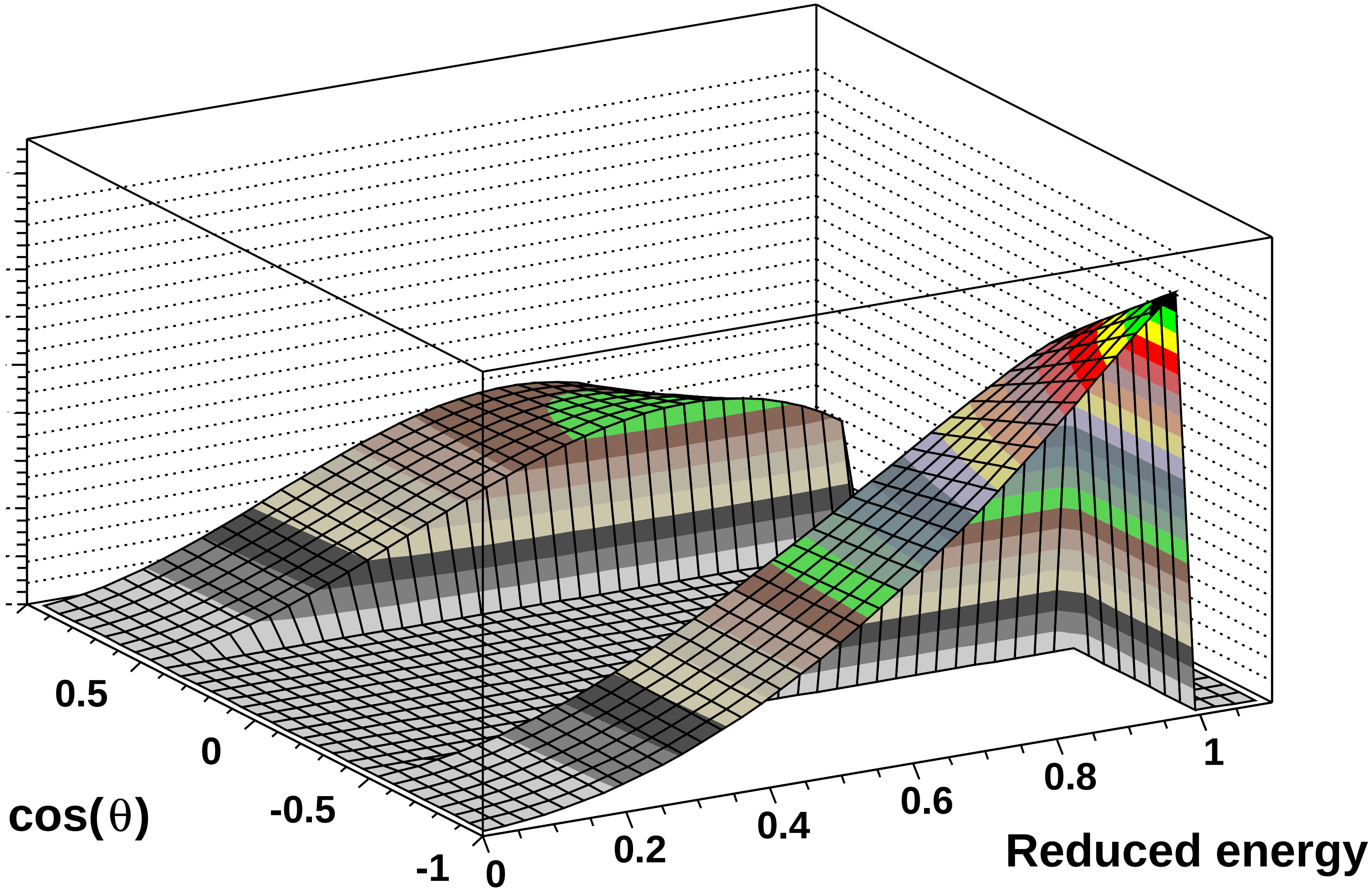
on behalf of the *TWIST* collaboration.

- Energy scale in *TWIST*
- Sensitivities of Michel parameters
- Calibration method
- Implementation & results
- Conclusion

Michel spectrum.

The middle region can not be seen by $\mathcal{TWIST}$.



Energy scale in $\mathcal{TWIST}$

- Energy scale β: defines a distortion of the reconstructed spectrum in the form

$$E \quad \longrightarrow \quad (1+\beta)\,E$$

- Upstream and downstream parts of the detector will be calibrated separately $\Rightarrow$ β_{up} and β_{dn}.
- Systematic error on a Michel parameter due to energy scale:

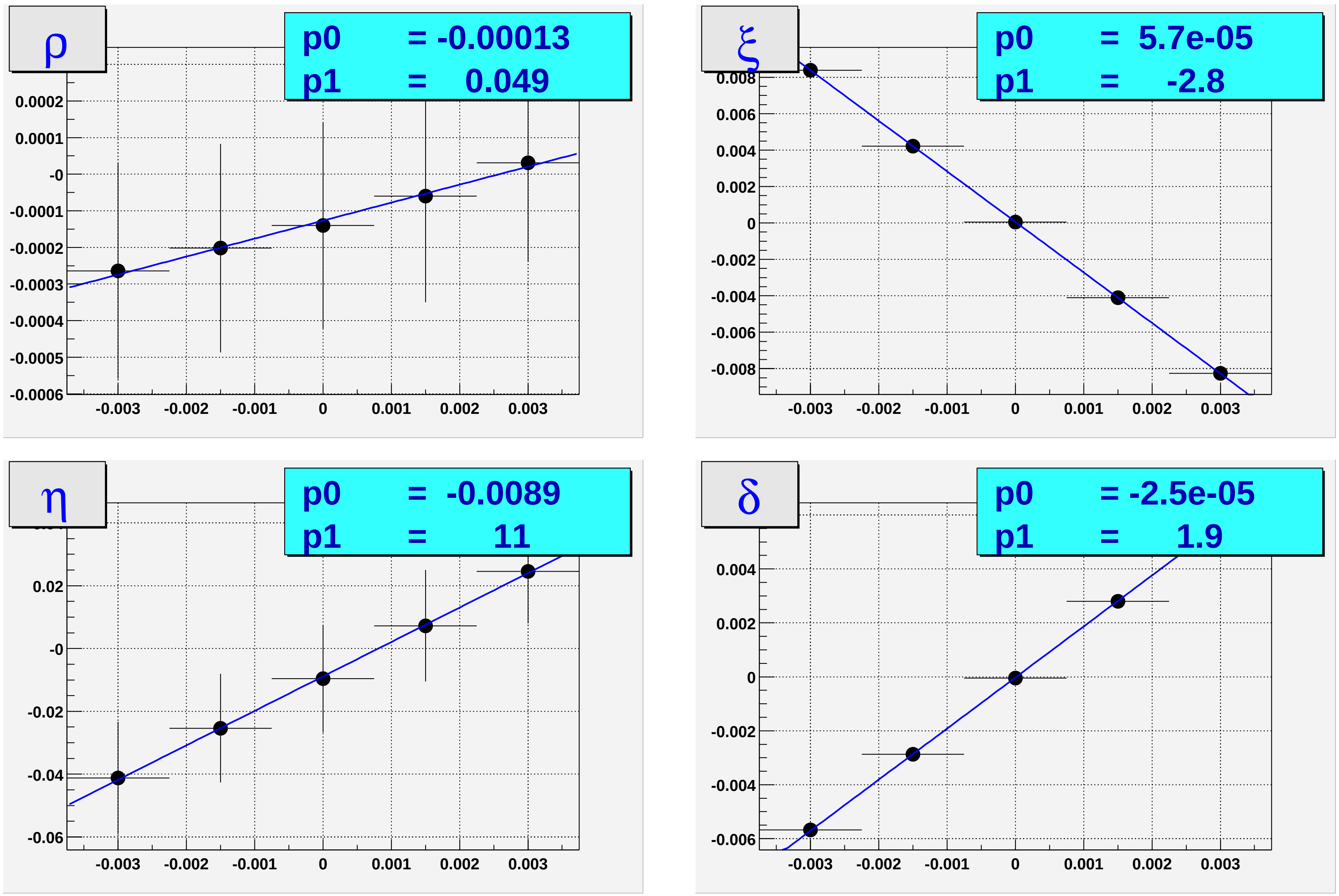
$$\Delta\rho \quad = \quad \frac{\partial\rho}{\partial\beta}\,\Delta\beta$$

  - ▷ Sensitivity $\partial\rho/\partial\beta = ?$
  - ▷ How well can we measure β?

Sensitivity of Michel parameters to energy scale

- Method: generate a MC Michel spectrum. Fit it with a distorted function fixing different β_{up} and β_{dn}
  $\rightarrow$ $\partial\rho/\partial\beta_{up}$, $\partial\rho/\partial\beta_{dn}$, $\partial\eta/\partial\beta_{up}$, $\partial\eta/\partial\beta_{dn}$ ...
- Realization:
  - ▷ 10^9 decays of 100% polarized muons simulated.
  - ▷ First order radiative correction included in the generation and in the fitting.
  - ▷ Log likelihood fit for 4 Michel parameters and normalization in the upstream and downstream regions simultaneously. Fit region is
    $[0.4 \leq x \leq 0.97] \times [0.5 \leq |\cos(\theta)| \leq 0.98]$
  - ▷ Distortion: 5 steps in β_{up} between $-30 \cdot 10^{-4}$ and $+30 \cdot 10^{-4}$.
    Same for the β_{dn}.

Deviations in Michel parameters vs β_{up}



Sensitivities—the result

For the fit region

$[0.4 \leq x \leq 0.97] \times [0.5 \leq |\cos(\theta)| \leq 0.98]$:

	$\frac{\partial}{\partial \beta_{up}}$	$\frac{\partial}{\partial \beta_{dn}}$
ρ	0.05	1.
η	11.	-1.9
ξ	-2.8	-0.9
δ	1.9	-1.

The energy calibration method

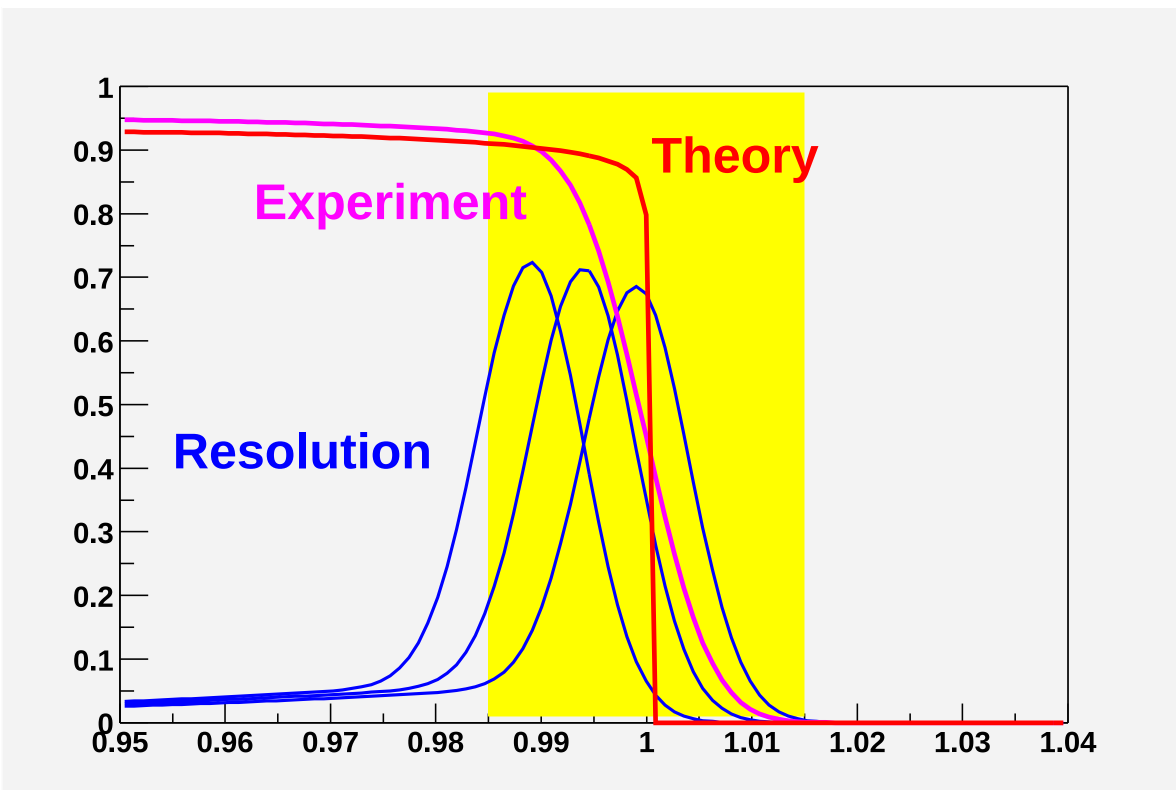
- The sharp edge of the Michel spectrum at the upper kinematic limit provides a natural calibration point.
- Positron energy loss affects the position of the reconstructed spectrum edge along with the energy scale.
- The planar detector geometry is essential. For that geometry the following equation for the edge of reconstructed spectrum is rigorously valid:

$$E_{edge} = (1+\beta)\left(E_{max} - \frac{\alpha}{|\cos(\theta)|}\right)$$

- Having determined E_{edge} for different angles we can find the constants α and β by fitting the equation.

Fitting the end point—1

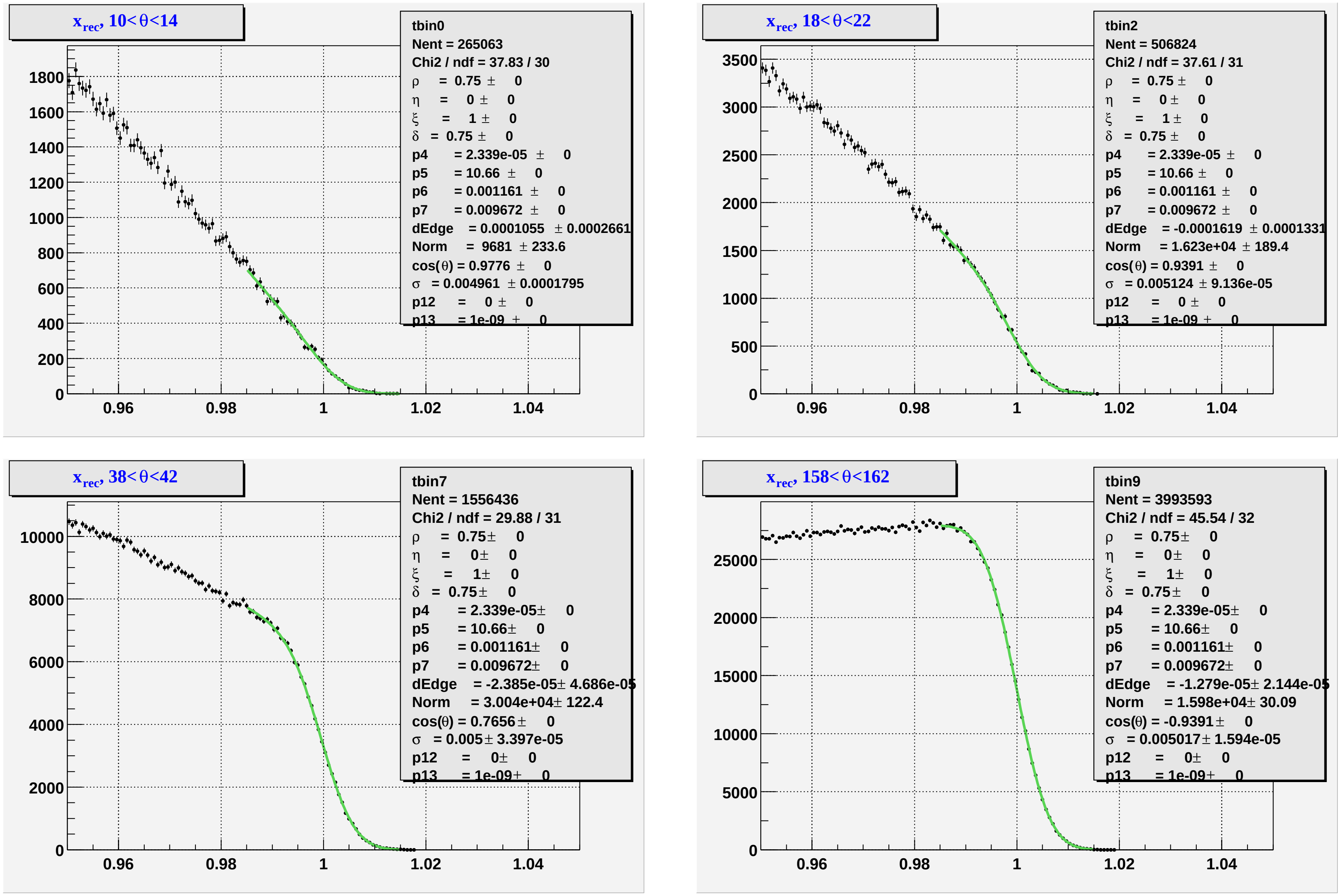
- Shape of the reconstructed spectrum edge is mainly defined by the resolution function.
- Approximation: theoretical Michel spectrum convoluted with a Gaussian.
  - ▷ No analytical expression available.



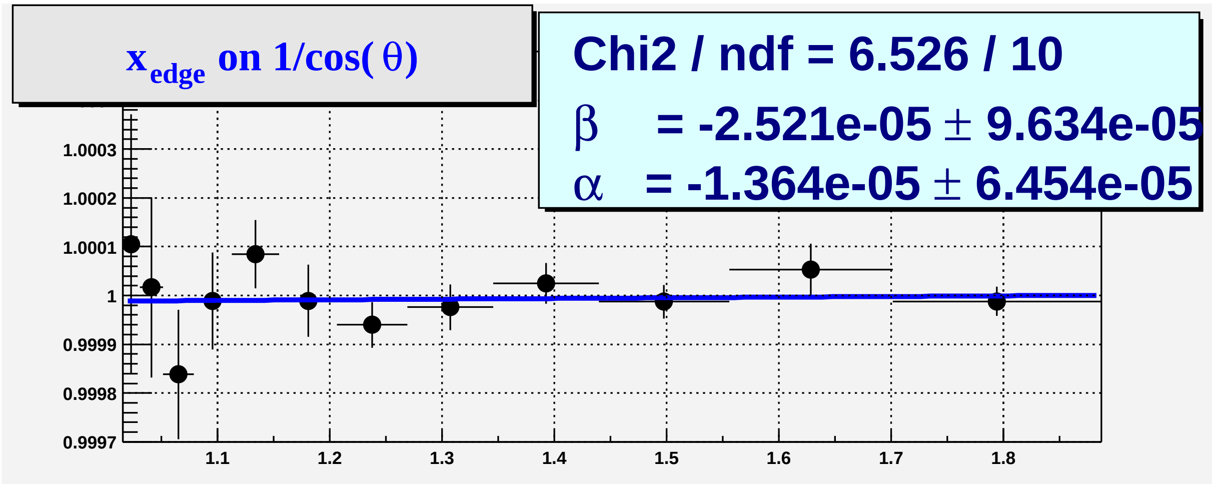
Fitting the end point—2

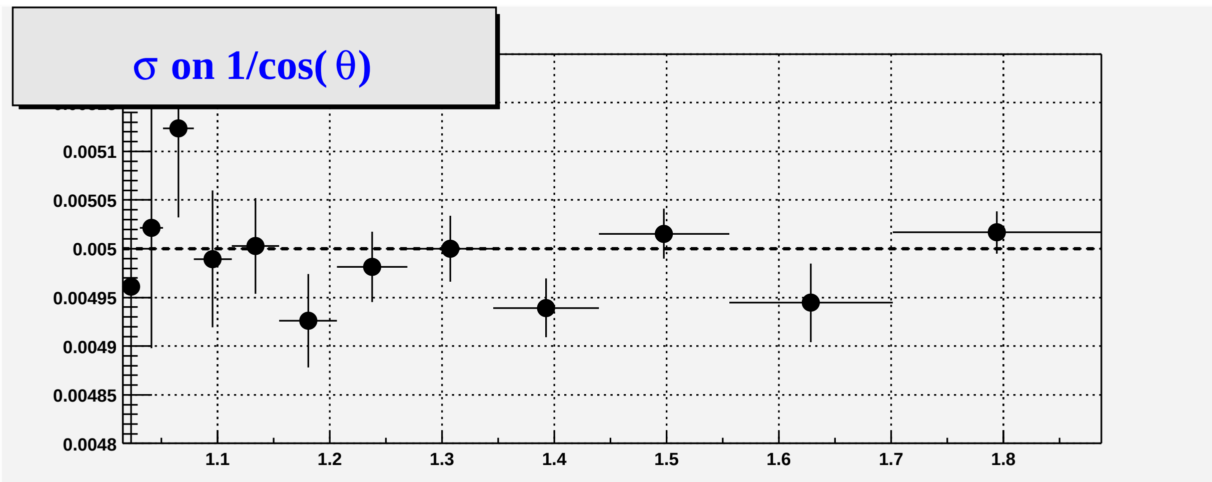
- Implementation:
  - ▷ Log likelihood fits with 3 free parameters: E_{edge}, σ and a normalization.
  - ▷ $\cos(\theta)$ for an angular bin $[\theta_1, \theta_2]$ is fixed at the mean value $(\cos(\theta_1) + \cos(\theta_2))/2$
- Testing:
  - ▷ Data sample corresponding to 10^9 total decays of 100% polarized muons.
  - ▷ Generated energy smeared with a Gaussian, $\sigma = 0.005$ (0.26 MeV)
  - ▷ Energy distributions in 4° angular bins between 10° and 58° and symmetrically upstream were produced and fitted with the convolution.

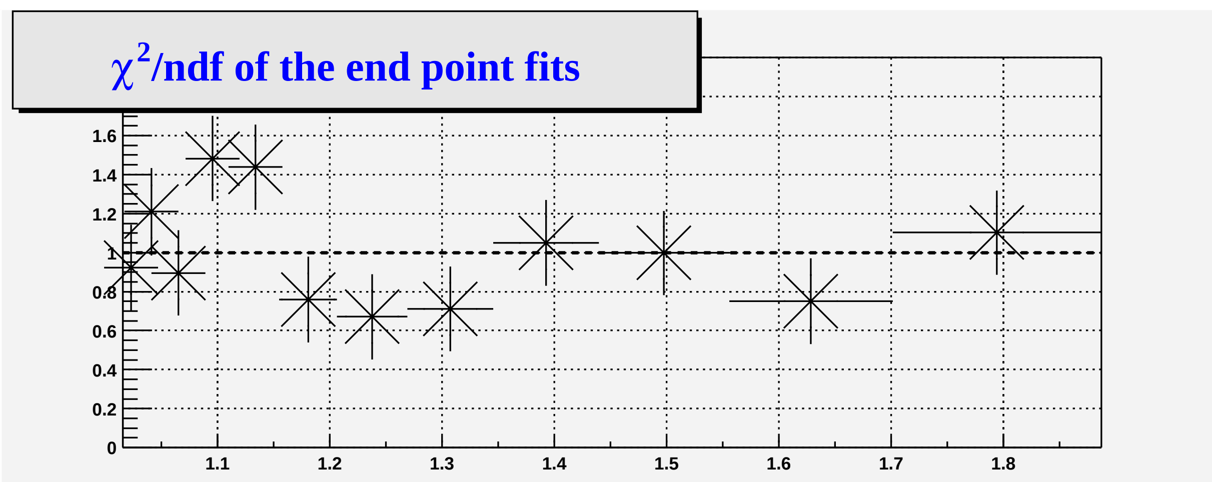
End point fit examples



Downstream energy calibration

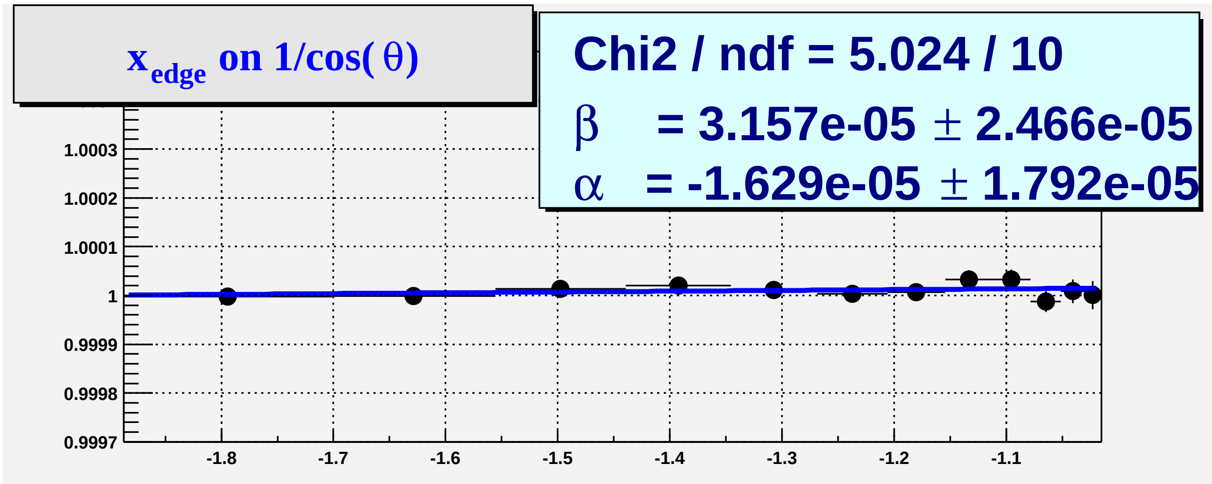


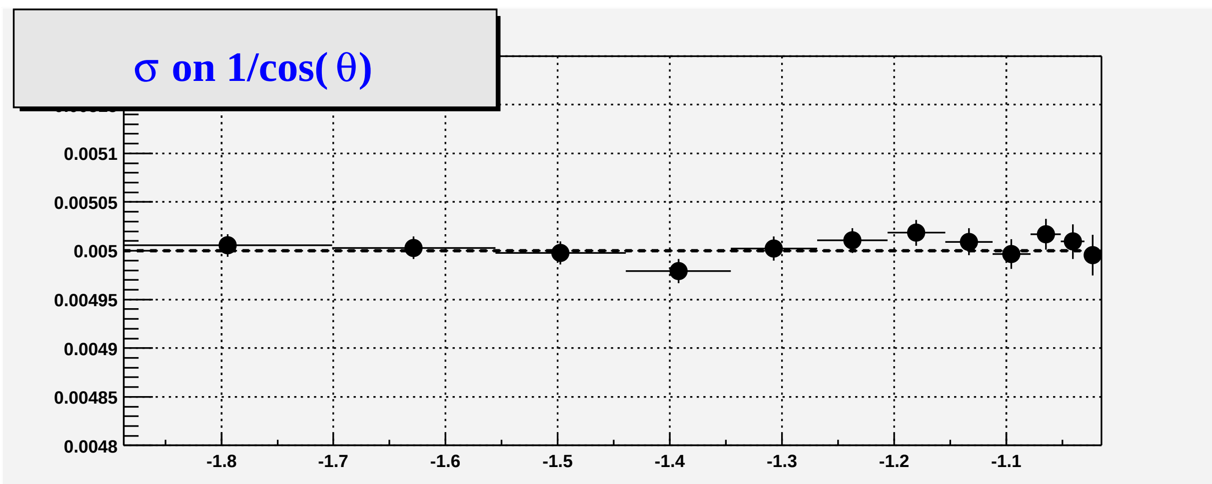


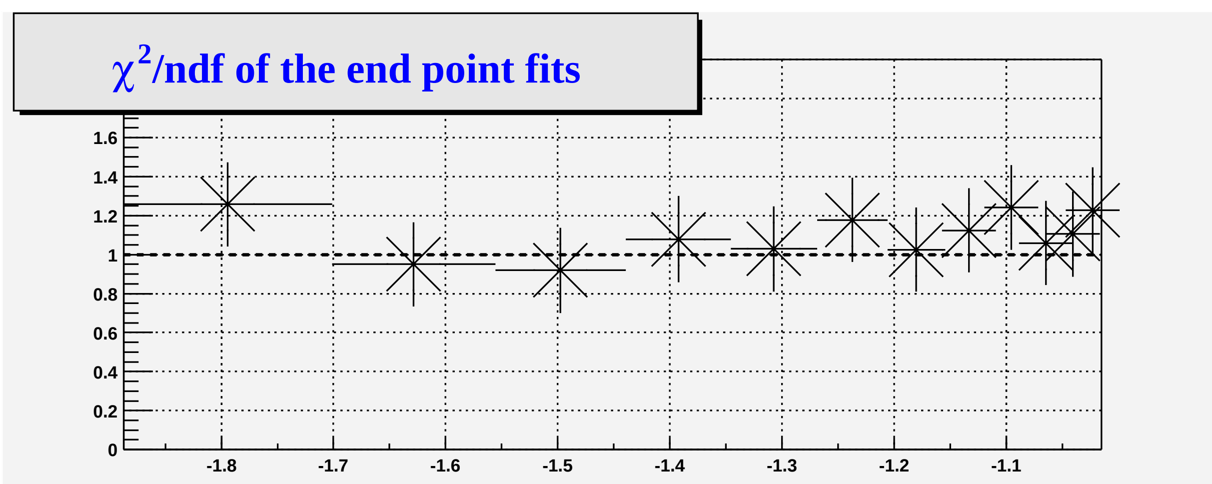


Horizontal scale is $1/\cos(\theta)$.

Upstream energy calibration







Horizontal scale is $1/\cos(\theta)$.

Energy calibration summary

- Precision of the upstream energy scale fit β_{up} is $0.25 \cdot 10^{-4}$, for β_{dn} it's $0.96 \cdot 10^{-4}$. The numbers were obtained assuming that the polarization of the decaying muons is perfect.
- Actual data set in addition to the surface muons contains also cloud muons. Not doing the RF cut brings in the analysis 5% of muons with the opposite polarization.
- For that subsample $\beta_{dn} = \sqrt{\frac{1}{0.05}} \times 0.25 \cdot 10^{-4} \approx 1.1 \cdot 10^{-4}$.
- Intrinsic to the data set energy calibration which combines the surface and the cloud muons gives precision for β_{up} $\pm 0.25 \cdot 10^{-4}$ and for β_{dn} $\pm 0.72 \cdot 10^{-4}$.

Conclusion

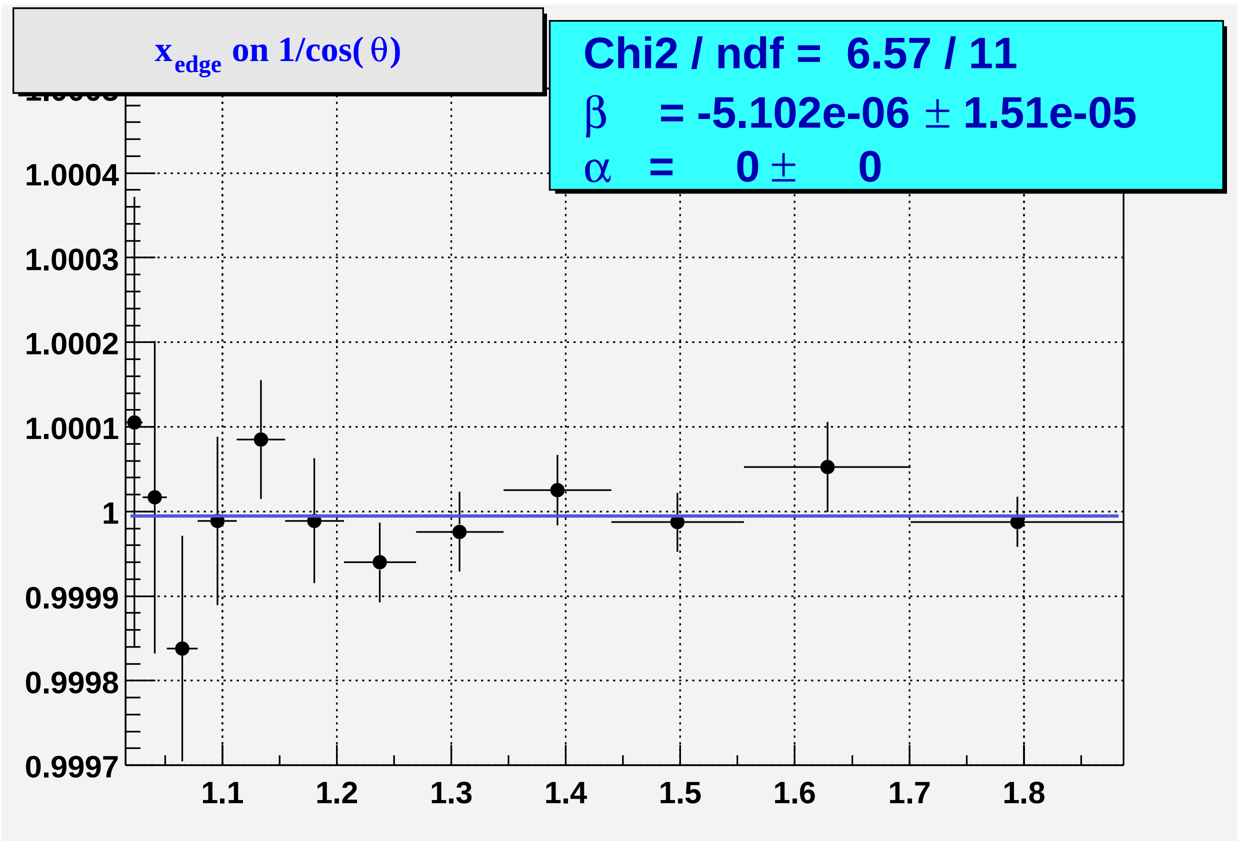
- Sharp edge of the Michel spectrum at the upper kinematic limit provides a natural calibration point.
- Planar detector design makes possible an exact account of the edge shift due to the positron energy loss.
- A calibration can be obtained which is intrinsic to the physics data sample. Systematic errors due to the e-scale uncertainty with that calibration are (in 10^{-4} units):

  $\Delta\rho = \pm 0.7$ $\quad$ $\Delta\eta = \pm 3.$

  $\Delta\xi = \pm 1.$ $\quad$ $\Delta\delta = \pm 0.9$
- In addition, a better calibration can be obtained by taking data using a beam with low muon polarization.

Additional slides.

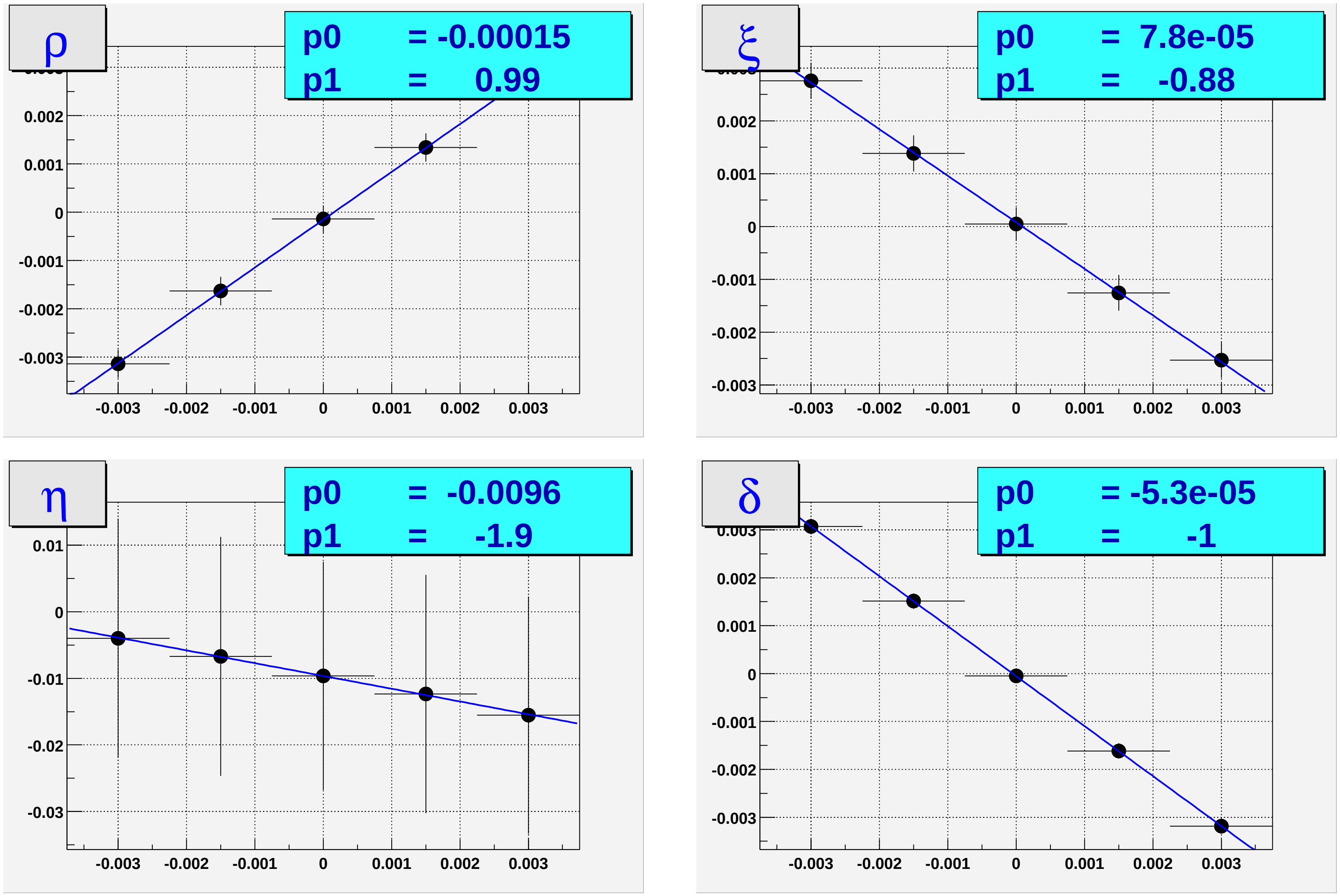
Downstream calibration with fixed α



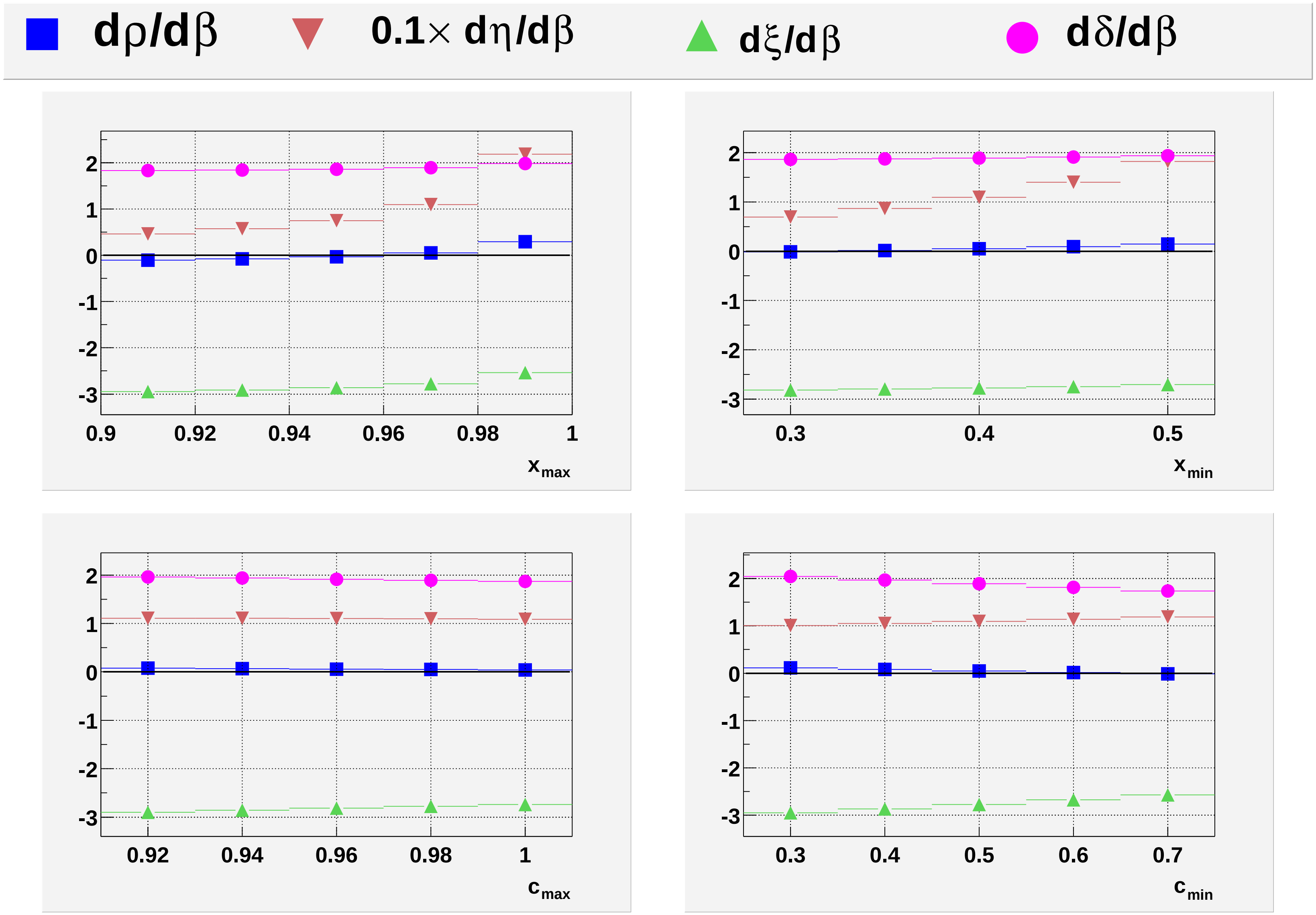
Amount of material is fixed: $\alpha_{up} + \alpha_{down} = \text{const}$

- Make a measurement of the sum in a dedicated run with low muon polarization.
- For physics data:
  - Fit α_{up}, β_{up}.
  - Fix α_{down} at a known value and do a single parameter fit for β_{dn} $\Rightarrow$ much smaller error.

Deviations in Michel parameters vs β_{dn}



Dependence of sensitivities to β_{up} on fit region



Dependence of sensitivities to β_{dn} on fit region